Chapter 3

Force

Before we do any real dynamics, we need to define what a force is. I know that you've already taken at least one physics course and a statics course. So, I know that you have worked with forces before. In this chapter, I cover some of the aspects of forces that students have most difficulty with. Furthermore, I provide a reference for the different types of elementary mechanics forces that we'll encounter in this course.

3.1 Definition: Force

To put it simply, ***a force is something that pushes or pulls on an object.*** Force has magnitude: the push/pull can be "weak" or "strong." Force also has direction: it can push/pull to the left, right, up, down, or any combination of these. Therefore, *force is a vector.* Force is often measured in units of pounds or Newtons. (1.0 N = 0.2248 lb.; or 1.0 lb = 4.448 N.)

You may recall from your Engineering Statics course that the first thing we do when we encounter a system of forces is we draw a free body diagram and depict the forces graphically. So let's take an example case that could have been in your statics course. Suppose a "snow bike" is parked on a hill as depicted in Figure 3.1. The front skid is free to slide. However, since the track in back is not moving, the snow bike does not slide down the hill.

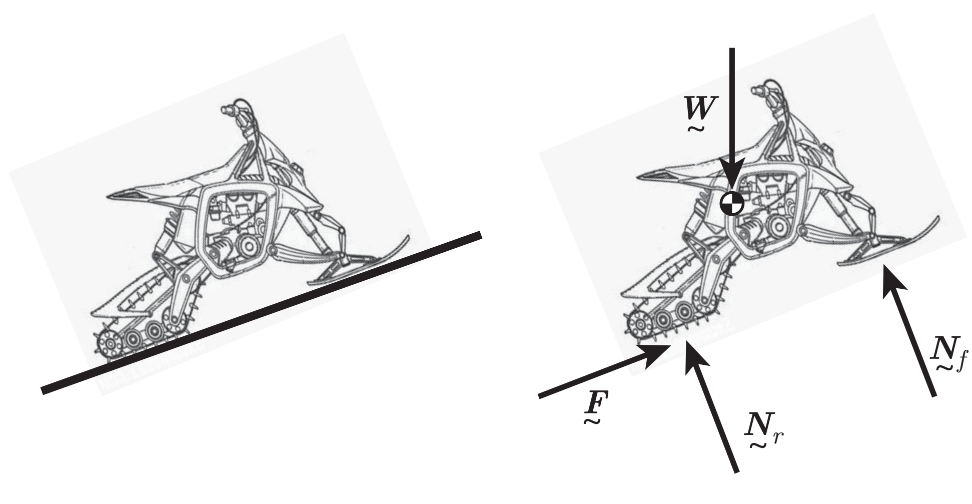

Figure 3.1: A "snow bike" parked on a hill, along with a free body diagram depicting forces acting on the vehicle.

A free body diagram showing all external forces acting on the bike is presented on the right side of the figure. Here $\underset{\sim}{\boldsymbol{W}}$ represents the weight of the bike, acting at the center of gravity. $\underset{\sim}{\boldsymbol{F}}$ is the friction on the track that prevents it from sliding backward down the hill. $\underset{\sim}{\boldsymbol{N}}_f$ and $\underset{\sim}{\boldsymbol{N}}_r$ are the normal forces acting on the front and rear of the bike, where they contact the hill.

In general, there are two different types of forces that can act on an object. The first type are *forces due to direct contact.* For example, the snow bike of Figure 3.1 is in direct contact with the the hill. The hill pushes and pulls on the bike. The hill provides the two normal forces and the friction force.

The other type of forces are ones that acts at a distance. Gravity (weight) is the perfect example of this type of force. The force $\underset{\sim}{\boldsymbol{W}}$ exists because the earth's gravity is pulling downward on the snow bike. In this example the bike happens to be in contact with the earth, but the weight doesn't need the contact to exist. If the bike happened to be flying through the air, it would still feel the gravitational pull of earth. There are other examples of forces that act over a distance[1], but the only one we'll consider in this class is gravity.

In the remaining sections of this chapter, I briefly discuss the types of forces we will encounter in this class.

3.2 Sources of Forces

What causes forces to act on an object? In this section we list the most common forces that we will encounter in Engineering Dynamics.

3.2.1 Gravity

In Newtonian Physics, there is an attractive force called gravity that acts between any two objects that have mass. Yes, *any* two objects. If you put two coins in your hand, for example, there are forces pulling the coins together.

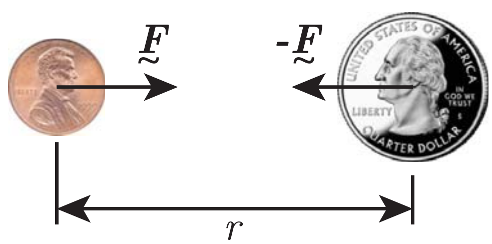

Figure 3.2: Gravitation force acting between two bodies.

The magnitude of the gravitational force is

$$\boxed{F = \frac{G\, m_1\, m_2}{r^2},} \tag{3.1}$$

where m_1 and m_2 are the masses of the two objects, r is the distance between the centers of the objects, and the gravitational constant is given by

$$G = 6.67384 \times 10^{-11}\, \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \tag{3.2}$$

[1]Other forces that act over a distance include magnetic forces and forces on charged particle moving in an electric field.

A penny has a mass of 2.5 g, and a quarter has a mass of 5.67 g. If the centers of the two masses are 5 cm (1.97 inches) apart, then the gravitational force pulling the coins together is 3.378×10^{-13} N, which is equivalent to 8.51×10^{-14} lb. This is 0.00000000138% of the weight of the penny. The reason you don't actually feel the coins pulling toward each other is because the force is so weak!

The gravitational force becomes noticeable when one or both the masses becomes large. When I say "large," I'm thinking about something as big as a planet. We can "feel" weight and we can observe objects falling to the ground because the earth has a mass of 5.9742×10^{24} kg.

3.2.2 Gravity Near Earth's Surface

According to Wikipedia, the mean radius of the Earth is 6,378.1 kilometers[2]. We'll call this radius r_e. So now imagine holding an object of mass m at a height of one meter above the earth's average surface radius. Then, the gravitational force tugging down on the object is

$$F = m \frac{G\, m_e}{(r_e + 1\,\mathrm{m})^2},$$

where m_e is the mass of the Earth reported in the previous section and G is the gravitational constant. Substituting these numbers into the expression above, we get

$$F = m\,(9.8229\,\frac{\mathrm{m}}{\mathrm{s}^2}). \tag{3.3}$$

Now, let's imagine performing the same experiment on the 16th floor of the Holmes Student Center which is approximately 160 feet (48.8 meters). In this case, the gravitational force pulling down on the object is

$$F = m \frac{G\, m_e}{(r_e + 48.8\,\mathrm{m})^2} =, m\,(9.8228\,\frac{\mathrm{m}}{\mathrm{s}^2}). \tag{3.4}$$

If we perform the experiment on top of Mount Everest (29,035 feet = 8849.9 meters), we get

$$F = m \frac{G\, m_e}{(r_e + 8849.9\,\mathrm{m})^2} =, m\,(9.7957\,\frac{\mathrm{m}}{\mathrm{s}^2}). \tag{3.5}$$

As indicated by Equation (3.1), the magnitude of the force due to gravity varies with distance squared. However, as we travel between the altitude extremes on planet Earth, the numbers in parentheses in (3.3) through (3.5) varies surprisingly little, only 0.27%.

Therefore, when we study the dynamics of objects on or near the Earth's surface, we usually assume that the magnitude of the force is constant:

$$\boxed{F = m\,g.} \tag{3.6}$$

For g, it is common to take $g = 9.81\,\mathrm{m/s}^2$ or $g = 32.2\,\mathrm{ft/s}^2$. Notice that g has units of *length per time squared* (L/T^2), the same as acceleration. Sometimes we call g the acceleration due to gravity.

The force of gravity is directed toward the center of the Earth. We usually think of it as straight down.

[2] I have had this number corroborated by other scientific sources as well.

3.2.3 Springs

Certainly, you have seen a spring. Some springs act in tension. Some act in compression. Still others can work in either tension or compression. To the engineer, a spring provides some flexibility or pliability in a structure.

The key feature of a spring is that it provides a restoring force. If a spring gets stretched beyond its *natural length*, there are internal forces within the spring which try to pull it back to its natural length. Likewise, if a spring is shorter than its natural length, then it provides a force pushing its ends outward in effort restore the spring to its natural length. This is depicted schematically in Figure 3.3.

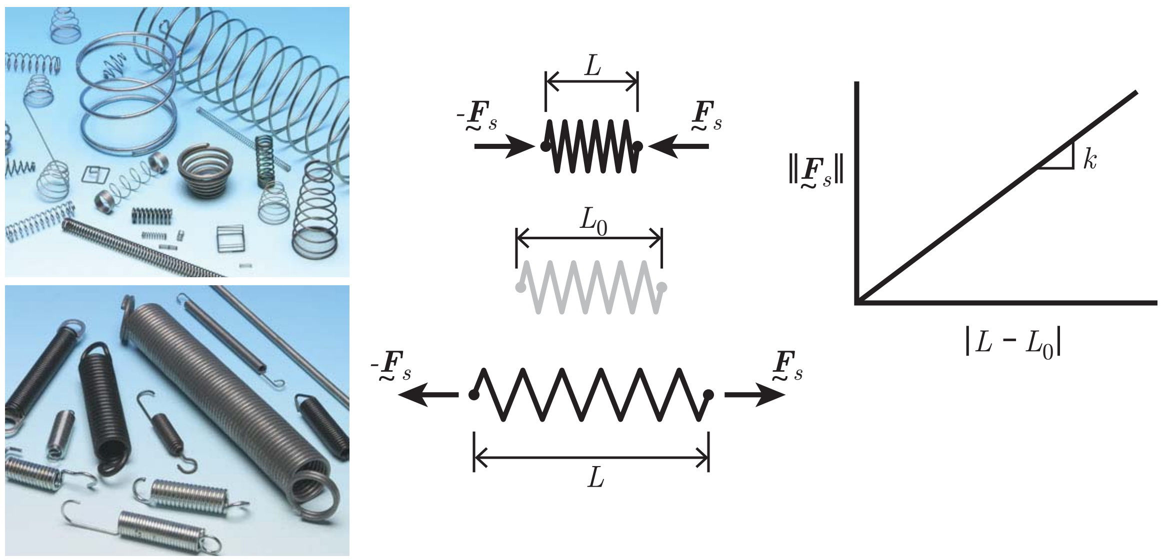

Figure 3.3: A linear model of a spring. The forces shown in the figure are the force acting on the springs. The forces of the springs acting on other bodies point in the opposite directions. (Photographs from Master Spring and Wire Form Co.)

The farther the spring is from its natural length, the more force the spring provides. If the spring is close to its natural length, the spring provides relatively little force. In the standard linear model of a spring, the relationship between spring force and the amount of stretch or compression of spring is that of a straight line:

$$\boxed{||\underset{\sim}{\boldsymbol{F}}_s|| = k\,|L - L_0|,} \tag{3.7}$$

where L is the actual length of the spring and L_0 is its natural length.

The coefficient k is called the "spring constant" or "spring rate." It has units of force per length (F/L). Graphically, one can think of the spring constant as the slope of the line in Figure 3.3.

It's important to know that (3.7) is just an approximation of how a spring behaves. The relationship between force and displacement in real springs is not perfectly linear, particularly for large displacements. However, for modest displacements, (3.7) is usually a good approximation.

3.2.4 Strings, Ropes, Cables, Cords

In our Engineering Dynamics course, strings, ropes, cables, and cords are all the same thing. That is, they have the same behavior.

This part is not written yet.

3.2.5 Bungee (Elastic) Cords

This part is not written yet.

3.2.6 Normal Force

Let me get one thing out of the way first.... When we say "normal force", the word "normal" means perpendicular. For some reason we professors neglect to tell students this; we assume that it's obvious. Then students end up thinking that "normal" means something equivalent to "usual" or "ordinary."

A nice example of normal forces are $\underset{\sim}{\boldsymbol{N}}_r$ and $\underset{\sim}{\boldsymbol{N}}_f$ shown in Figure 3.1. The figure shows a "snow bike" resting on a sloped surface. The two normal forces are *perpendicular* to the sloped surface. If those two forces weren't there, the snow bike would fall through the surface.

In general, a normal force plays the role of what we call a *constraint force.* It prevents the object from penetrating a solid surface. As such, the normal force usually acts in only one direction: it can "push" but not "pull." What do I mean by this? If we think about the normal forces $\underset{\sim}{\boldsymbol{N}}_r$ and $\underset{\sim}{\boldsymbol{N}}_f$ in Figure 3.1, then we say that the normal forces "push" the snow bike away from the sloped surface. The normal forces cannot pull the snow bike toward the surface.

To calculate the magnitude of the normal force, you have to solve a dynamics problem or a statics problem.

3.2.7 Friction

As described in the previous section, when two object touch each other, there is a normal force acting perpendicular to the surfaces which prevents the two objects from penetrating each other. Usually, there is also a force between the objects that acts tangential to the surfaces. This force either opposes or completely prevents the two surfaces from sliding relative to each other. The force is called *friction.*

What causes friction? In Figure 3.4, we show two surfaces in contact with each other. Although the surfaces may appear smooth to the naked eye, if you get a microscope and zoom in close enough, you'll notice irregularities. The surfaces have bumps and valleys.

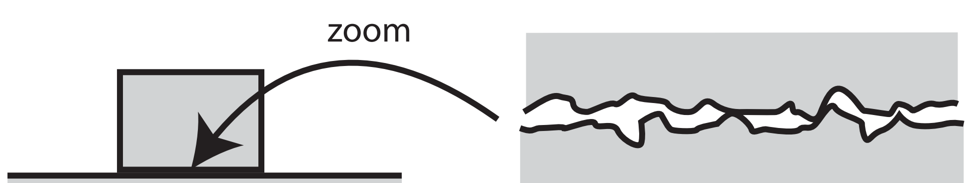

Figure 3.4: Friction. At a microscopic level, surfaces have bumps and valleys that get interlocked, hindering or preventing sliding motion.

At this small length scale, these bumps and valleys get interlocked with each other, making it difficult for the two surfaces to slide relative to each other. This is friction! If the surfaces do slide, little bits and pieces of the surfaces break off. This is called "wear."

Dependence on Normal Force.

If the force pressing the two surfaces together is large, you can probably imagine that the surfaces deform more. The surfaces touch at more places, and the bumps and valleys get pushed into each other more deeply. *Therefore, when the normal force is larger, the friction force which resists sliding generally gets stronger.*

Reducing Friction

The machines that engineers build often have parts that rub and slide against each other. Often, one would like to reduce friction as much as possible. One possible way to reduce friction is to build the two surfaces out of hard, polished materials. Polishing reduces the size of the bumps and valleys. Hardness, reduces the amount by which the surfaces deform into each other.

Another way to reduce friction is to lubricate the surfaces. A thin film of lubricant can fully or partially separate the surfaces, thus reducing the resistance to sliding motion. Furthermore, one may separate the surfaces mechanically by placing rollers or spherical balls between the the surfaces.

Although one has to go to extraordinary measures to completely eliminate friction, one can obtain situations for which the the friction is sufficiently small that we can ignore it. From an engineering analysis perspective, this is good because friction is a complex process of bumps and valleys bumping into each other that is often difficult to account for analytically.

Two Types of Friction

In this Engineering Dynamics class, we'll classify friction as coming in two different types: *static friction* and *kinetic friction.* The distinction is fairly straightforward. If the two surfaces in contact are actually sliding relative to each other, we say the tangential force resisting the sliding is kinetic friction. If the two objects are *not* sliding relative to each other, then the tangential force which prevents sliding is static friction.

As an example to refer to, Figure 3.5a shows a box sliding down a ramp. The tangential force from the ramp onto the box, is *kinetic friction.* The kinetic friction force on the box is directed upward and to the left and is labeled $\underset{\sim}{\boldsymbol{F}}_k$.

If the box happens to be perched on the ramp without sliding as shown in Figure 3.5b, then the tangential force preventing the box from sliding is *static friction.* It is labeled $\underset{\sim}{\boldsymbol{F}}_s$ in the figure.

In Figure 3.5c, a conveyor belt is pulling the box upward and to the left at a constant speed. What do you think? Is kinetic friction or static friction acting on the box?

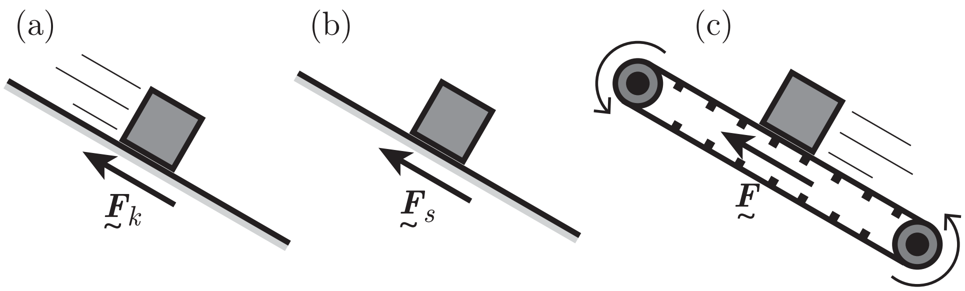

Figure 3.5: Three examples of two different types of friction.

If the conveyor belt and the box are moving with the same speed, then the box is *not sliding* on the belt. Therefore, $\underset{\sim}{\boldsymbol{F}}$ in the figure is the result of static friction. Yes, the box is moving. However, the important point to recognize is that the box is not sliding.

3.2.8 Static Friction

Here's a **mistake** that many, many, many students make. They say that the magnitude of the static friction force is $\mu_s N$, where μ_s is the coefficient of static friction and N is the normal force.

To illustrate why this is a mistake, consider the following thought experiment in which a big stainless steel pot, filled with ramen noodles, is resting on a wooden table. As shown in Figure 3.6, we wrap a string around the pot and pull to the right with a force $\underset{\sim}{\boldsymbol{T}}$.

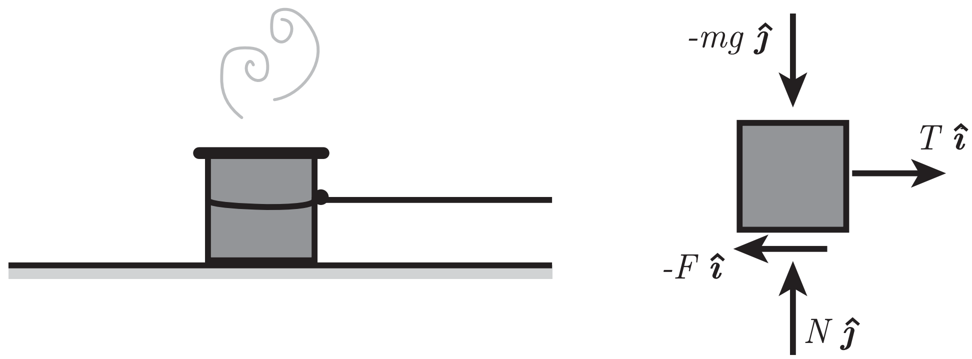

Figure 3.6: A simple friction thought experiment.

To begin our experiment, we'll pull with a force of 1 pound: $\underset{\sim}{\boldsymbol{T}} = 1.0\,\mathrm{lb}\,\hat{\boldsymbol{\imath}}$. It's a small force. And with that small pull to the right, the pot of noodles doesn't budge. It remains stationary.

On the right side of Figure 3.6, I present a free body diagram of the pot. Since the pot is not moving, we have a statics problem that you should be able to solve in your sleep. In particular, the equilibrium equations are obtained by summing all the forces and setting them to zero. In the horizontal ($\hat{\boldsymbol{\imath}}$) direction, we find

$$\hat{\boldsymbol{\imath}}: \qquad T - F = 0, \tag{3.8}$$

In the vertical ($\hat{\boldsymbol{\jmath}}$) direction, we obtain

$$\hat{\boldsymbol{\jmath}}: \qquad N - mg = 0. \tag{3.9}$$

The second equation above tells us that the normal force is equal to the weight: $N = mg$. But this doesn't matter. Regardless of the normal force, as long as the pot is not moving, Equation (3.8) tells us that $F = T$. The friction force is equal to the tension force in the string.

Therefore, in our thought experiment with $T = 1.0\,\mathrm{lb}$, our static analysis tells us that the static friction force is $F = 1.0\,\mathrm{lb}$.

Now let's suppose we run out thought experiment again, keeping everything the same, except this time we'll pull on the pot with $T = 2\,\mathrm{lb}$. Suppose, again, that the pot does not move. Because of (3.8) the friction force must be $2\,\mathrm{lb}$.

In the case of static friction, the *friction force is whatever is required to keep the object in equilibrium.* The friction force, generally, is **not** $\mu_s N$.

The Limits of Static Equilibrium

Returning to our thought experiment, imagine that I continue to re-run the experiment with $T = 3\,\mathrm{lb}$; then $T = 4\,\mathrm{lb}$. Each time I re-run the experiment, I increase T. As long as the string is sufficiently strong, I will eventually reach a tension for which the pot of noodles begins sliding along the table.

Another way of saying this is that there is a maximum tension $T = T_{max}$ at which the pot will remain in static equilibrium. Any tension above T_{max} will cause the pot to slide. Given that $F = T$ when the system is in static equilibrium, we can also say there is a maximum static friction force F_{max} that can hold the system in equilibrium.

The Coulomb Model

Now here's a question, I'd like you to think about... Suppose we have two identical, huge pots. One is filled to the top with wet, heavy ramen noodles. The other pot is nearly empty. Which of the pots would have a bigger F_{max}?

Based upon your every-day experiences with the world and how it works, I suspect that most of you would guess that the heavier pot would have a bigger F_{max}. If you ran a *real* experiment, collecting *real* data, you would find out that this is true.

Often, one finds the relationship between F_{max} and the magnitude of the normal force N pushing the surfaces together to be approximately linear as illustrated in Figure 3.7. The reason why a larger normal force corresponds to a larger F_{max} was discussed in Section 3.2.7. The bumps and valleys on the surfaces get pushed into each other more, creating more direct contact, making it better able to resist sliding.

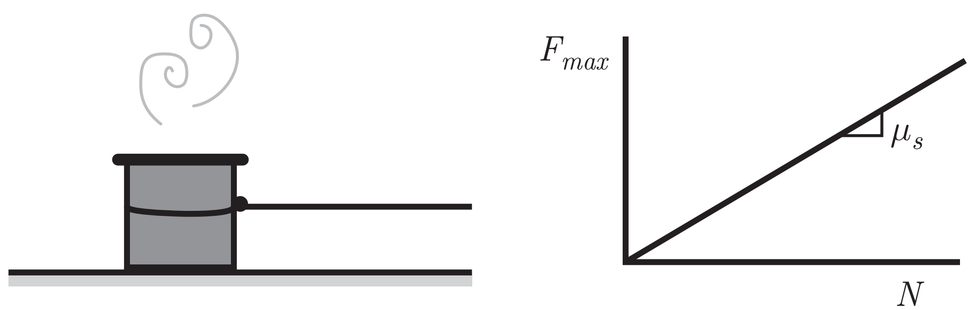

Figure 3.7: The Coulomb model of static friction.

The fact that the relationship between F_{max} and N is approximately linear is nice because it allows us engineers to take a very complicated phenomenon (friction) and represent it by a simple mathematical expression:

$$\boxed{F_{max} = \mu_s N, \qquad \text{or} \qquad F \leq \mu_s N.} \tag{3.10}$$

The equation above is called the Coulomb model of static friction. It is a model or rule that states that maximum static friction F_{max} is strongly dependent on the normal force pushing the two surfaces together. The slope of the line in Figure 3.7 is called the coefficient of static friction μ_s. Note that it is a ratio of two forces, so μ_s is unitless. The value of μ_s depends on the types of materials the two surfaces are made of, as well as their roughness.

Also note that the Coulomb model does not include dependence on other seemingly important parameters such as contact area. It turns out that contact area is not important. The reason for this is a bit tricky, so I will not attempt to explain it here.

3.2.9 Kinetic Friction

Kinetic friction, by definition, is dynamic. Therefore, one might be tempted to think that kinetic friction is more difficult to work with than static friction. Actually, it's easier.

When two surfaces are sliding relative to each other, the Coulomb Kinetic Friction Model simply states that the magnitude of the friction is equal to

$$\boxed{||\underset{\sim}{\boldsymbol{F}}|| = \mu_k\, N.} \tag{3.11}$$

Here, N is the magnitude of the normal force, and μ_k is the coefficient of kinetic friction. Like the coefficient of static friction, μ_k is unitless and it depends on the types of surfaces in contact. It turns out that $\mu_k \leq \mu_s$ for any pair of surfaces. In Section ???, we'll discuss why this is true.

What makes kinetic friction "easier" than static friction is that (3.11) has an equals sign. As soon as you know the normal force, you can determine the friction force. In contrast, the static friction model (3.10) has an inequality ($\leq$); static friction is determined by static or dynamic analysis.

When analyzing problems with friction, it is important to get the direction of the friction correct. The friction force always points tangent to the surface. Its direction is such that it opposes the relative motion of the surfaces.

Finally, I think it's interesting to make note of what is *not* in the Coulomb friction model. Like static friction, the kinetic friction model does not have dependence on contact surface area. This means that contact over a larger surface area produces the same force as contact over a smaller area. Also interesting (perhaps surprising) is the fact that the model does not account for slip speed. Objects sliding across each other quickly produce the same friction as objects sliding slowly. Spending some time playing with a belt sander might convince you that this is empirically true.

3.3 Take-Aways

Here are the Take-Aways for Chapter 3

1. A *force* is something that pushes or pulls on an object.
2. Force is a *vector*; it has both magnitude and direction.
3. Force is often measured in pounds (lb) or Newtons (N). However, in Section ???, we'll see that force can also be expressed in terms of mass, length, and time.
4. Most forces (excluding gravity) that act on a body do so by direct physical contact.
5. Gravity is an exception to the take-away above; it acts at a distance.
6. When an object is within a few tens of thousands of feet of the Earth's surface, the gravitational pull is essentially constant.
$$\underset{\sim}{\boldsymbol{F}} = -mg\,\hat{\boldsymbol{\jmath}},$$
where $-\hat{\boldsymbol{\jmath}}$ is the downward direction.
7. If you consider objects in orbit (e.g. satellites, moons, other planets), then you should use the full Newtonian, nonlinear form of the gravitational force:
$$||\underset{\sim}{\boldsymbol{F}}|| = \frac{G\, m_1\, m_2}{r^2}.$$
8. A spring provides a "restoring force" which tries push or pull it back to its natural length, L_0. According the the standard linear model, the magnitude of the spring force is given by
$$||\underset{\sim}{\boldsymbol{F}}_s|| = k\,|L - L_0|,$$
where L is the actual length of the spring and k is the spring constant.

9. The spring constant has units of *force per length* (F/L).

10. A *Normal Force* acts perpendicular to a surface. It prevents a body from penetrating a solid surface. Usually a normal force only "pushes"; it doesn't pull. To calculate the magnitude of a normal force, you must solve a dynamic or statics problem.

11. *Static friction* occurs when a tangential friction force between objects is sufficiently strong that it prevents the two surfaces from sliding relative to each other.

    (a) In the case of static friction, the amount of friction is whatever is necessary to prevent the prevent the surfaces from sliding.

    (b) In order for a static friction force to prevent surfaces from sliding, the magnitude must be less than the coefficient of static friction times the magnitude of the normal force: $F < \mu_s N$. This is the Coulomb model for static friction.

12. *Kinetic friction* occurs when two surfaces are sliding relative to each other. According to the Coulomb model, kinetic friction has magnitude $||\underset{\sim}{\boldsymbol{F}}|| = \mu_k N$. The direction of the friction force is tangent to the surfaces and opposes the sliding action.

13. In both the static and kinetic Coulomb friction models, the coefficients μ_s and μ_k are unitless. They depend on the materials of the two surfaces in contact. In general $\mu_k \leq \mu_s$.

14. Interestingly, the Coulomb models do not have any dependence on parameters such as contact area. The kinetic model does not depend on the speed at which the surfaces are sliding.