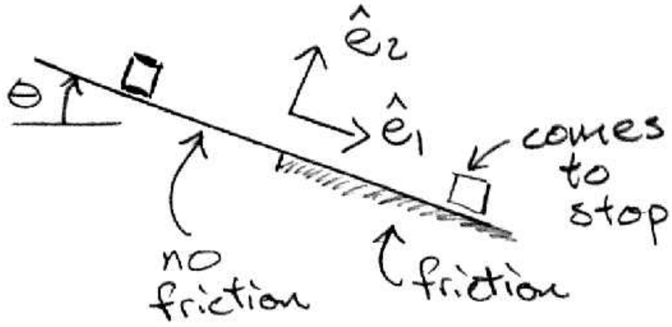


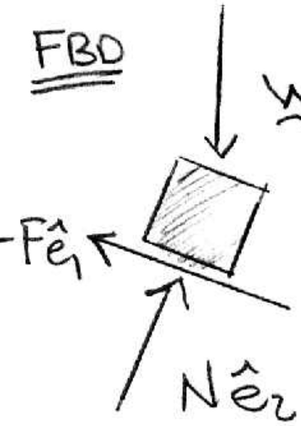
Given: $m, \theta, \mu_k, \mu_s, g$

Initial speed $v_0 = 0$

Time to reach friction $t = t_f$

Find: Time t_s to come to stop

FBD



$$\underset{\sim}{W} = mg\sin(\theta)\hat{e}_1 - mg\cos(\theta)\hat{e}_2$$

MAD

Newton $\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$$\hat{e}_1:\quad mg\sin(\theta) - F = ma \qquad (1)$$

$$\hat{e}_2:\quad N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$$

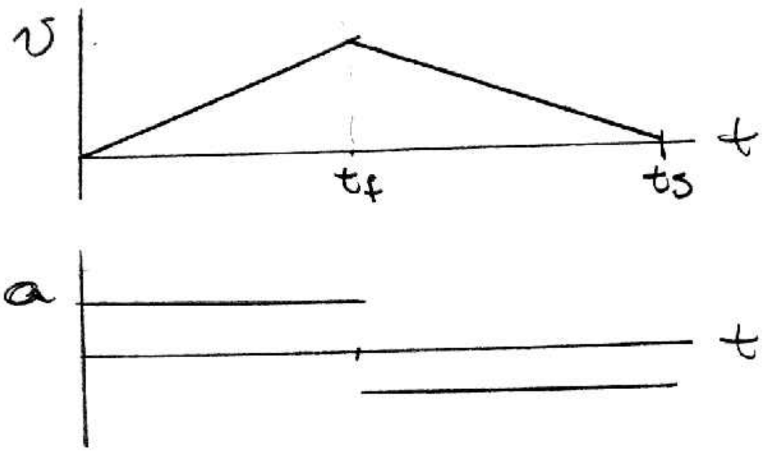
Before Friction:

$$a = g\sin(\theta) \qquad (*)$$

With Friction

$$a = g\sin(\theta) - F = g\sin(\theta) - \mu_k N$$

$$a = g\sin(\theta) - \mu_k g\cos(\theta) \qquad (\#)$$



Define velocity
$\underset{\sim}{v} = v\hat{e}_1$

Kinematics:

- Before friction $0 < t < t_f$

$$v = \int a\,dt = \int g\sin(\theta)\,dt = g\sin(\theta)t + c_1$$

Initial condition $v(0) = g\sin(\theta)\cdot 0 + c_1 = 0$

$$\Rightarrow c_1 = 0$$

$$\Rightarrow v(t) = g\sin(\theta)t \quad (\text{for } 0 \le t \le t_f)$$

$$v(t_f) = g\sin(\theta)t_f$$

- After Friction $t \ge t_f$

$$v = \int a\,dt = \int g\left(\sin(\theta) - \mu_k\cos(\theta)\right)dt$$

$$= g\left[\sin(\theta) - \mu_k\cos(\theta)\right]t + c_2$$

$$v(t_f) = g\left[\sin(\theta) - \mu_k\cos(\theta)\right]t_f + c_2 = g\sin(\theta)t_f$$

$$\Rightarrow c_2 = g\mu_k\cos(\theta)t_f$$

$$\Rightarrow v(t) = g\left[\sin(\theta) - \mu_k \cos(\theta)\right] t + \mu_k g \cos(\theta) t_f$$

Want to find what time this stops, $t = t_s$

$$v(t_s) = g\left[\sin(\theta) - \mu_k \cos(\theta)\right] t_s + \mu_k g \cos(\theta) t_f = 0$$

$$t_s = \frac{-\mu_k g \cos(\theta) t_f}{g\left[\sin(\theta) - \mu_k \cos(\theta)\right]}$$

$$\boxed{t_s = \frac{\mu_k \cos(\theta)}{\mu_k \cos(\theta) - \sin(\theta)} t_f}$$

Check Units $\frac{1}{1} T = T$ ✓