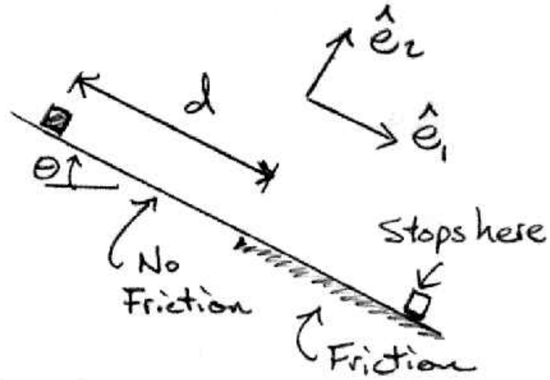
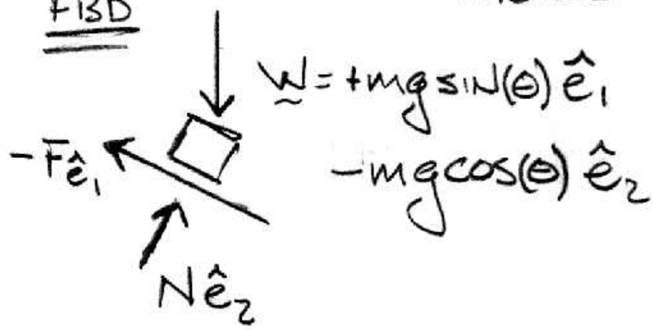


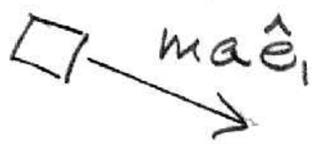
Given: $m, \theta, \mu_k, \mu_s, g, d$
Initial speed $v_0 = 0$

Final: Time t_s to come to stop

<u>FBD</u>



<u>MAD</u>



<u>Newton</u>

$\hat{e}_1$: $mg\sin(\theta) - F = ma$

$\hat{e}_2$: $N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$

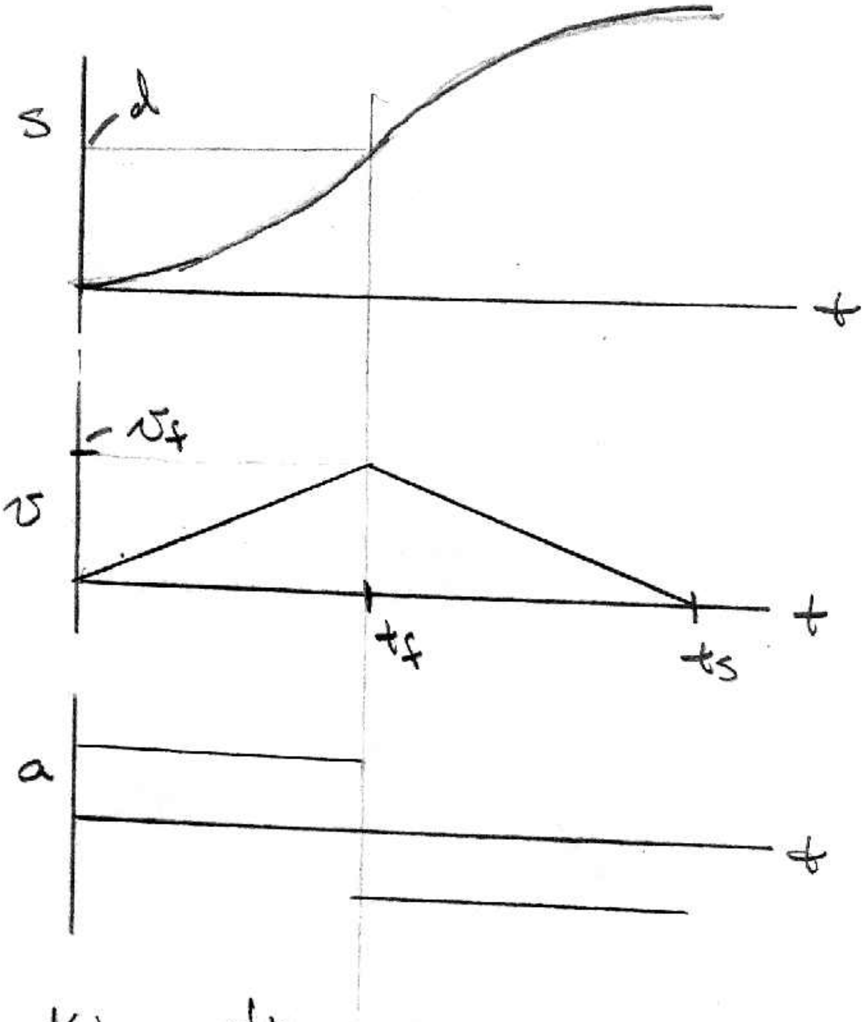
Before Friction: $0 \le t < t_f \leftarrow$ Time at which friction starts (Not known)

$$a = g\sin(\theta) \quad (1)$$

After friction begins: $t_f \le t \le t_s$

$$a = g\sin(\theta) - \frac{1}{m}F, \quad \text{but } F = \mu_k N = \mu_k mg\cos(\theta)$$

$$\Rightarrow a = g\sin(\theta) - \mu_k g\cos(\theta) \quad (2)$$



Write position as $\underset{\sim}{r} = s\hat{e}_1$

Write velocity as $\underset{\sim}{V} = v\hat{e}_1$

Kinematics:

For $0 \leq t < t_f$

$$v(t) = \int a\,dt = \int g\sin(\theta)\,dt = g\sin(\theta)t + c_1$$

I.C. $v(0) = g\sin(\theta)\,0 \cdot c_1 = 0 \quad \Rightarrow c_1 = 0$

$\Rightarrow v(t) = g\sin(\theta)\,t \quad$ for $0 \leq t < t_f \quad (\#)$

$$s(t) = \int v(t)\,dt = \int g\sin(\theta)\,t\,dt = \frac{1}{2}g\sin(\theta)\,t^2 + c_2$$

I.C. $s(0) = \frac{1}{2}g\sin(\theta)\cdot 0^2 + c_2 = 0 \Rightarrow c_2 = 0$

$$s(t) = \frac{1}{2}g\sin(\theta)\,t^2$$

Note: reaches friction when $S(t) = d$

$$S(t_f) = \frac{1}{2} g \sin(\theta)\, t_f^2 = d$$

$$\Rightarrow \quad t_f = \sqrt{\frac{2d}{g\sin(\theta)}} \quad (*)$$

Note that velocity when friction starts is

$$v(t_f) = g\sin(\theta)\, t_f = g\sin(\theta)\sqrt{\frac{2d}{g\sin(\theta)}} = \sqrt{2gd\sin(\theta)}$$

from (#)

Kinematics $\quad t_f \le t < t_s$

$$v(t) = \int a\, dt = \int g\left[\sin(\theta) - \mu_k\cos(\theta)\right] dt$$

$$= g\left[\sin(\theta) - \mu_k\cos(\theta)\right] t + C_3$$

Boundary Condition

$$v(t_f) = g\left[\sin(\theta) - \mu_k\cos(\theta)\right] t_f + C_3 = \sqrt{2gd\sin(\theta)}$$

(*)

or $\quad g\left[\sin(\theta) - \mu_k\cos(\theta)\right]\sqrt{\frac{2d}{g\sin(\theta)}} + C_3 = \sqrt{2gd\sin(\theta)}$

or $\quad \cancel{2gd\sin(\theta)} - \mu_k\cos(\theta)\sqrt{\frac{2gd}{\sin(\theta)}} + C_3 = \cancel{2gd\sin(\theta)}$

$$\Rightarrow C_3 = \mu_k \cos(\theta)\sqrt{\frac{2gd}{\sin(\theta)}}$$

So

$$v(t) = g\left[\sin(\theta) - \mu_k \cos(\theta)\right]t + \mu_k \cos(\theta)\sqrt{\frac{2gd}{\sin(\theta)}}$$

Stops at $t = t_s$

$$v(t_s) = g\left[\sin(\theta) - \mu_k \cos(\theta)\right]t_s + \mu_k \cos(\theta)\sqrt{\frac{2gd}{\sin(\theta)}} = 0$$

$$\Rightarrow t_s = \frac{\mu_k \cos(\theta)}{g\left[\mu_k \cos(\theta) - \sin(\theta)\right]}\sqrt{\frac{2gd}{\sin(\theta)}}$$

$$\Rightarrow \boxed{t_s = \frac{\mu_k \cos(\theta)}{\mu_k \cos(\theta) - \sin(\theta)}\sqrt{\frac{2d}{g\sin(\theta)}}}$$

Units $\frac{1}{1}\sqrt{\frac{L}{L/T^2}} = \sqrt{T^2} = T$ ✓