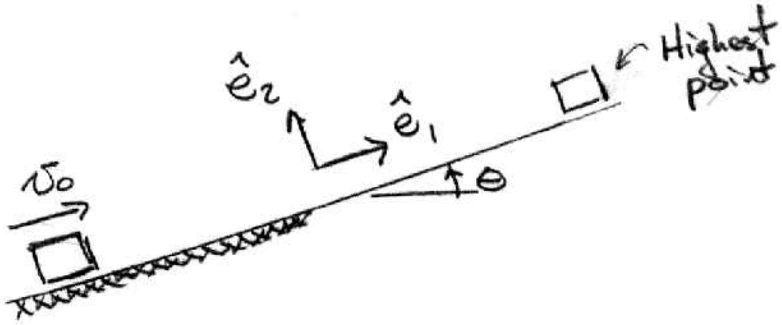
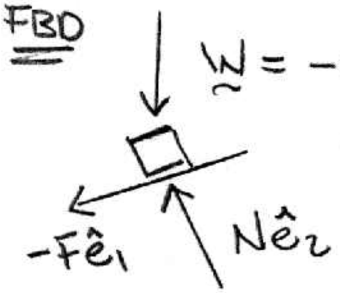


Given: $m, g, \theta, \mu_k, \mu_s$
v_0, t_f

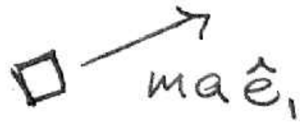
Find: t_s ← time it reaches highest point

FBD



$\underset{\sim}{W} = -mg\sin(\theta)\hat{e}_1 - mg\cos(\theta)\hat{e}_2$

MAD



expect $a<0$

Newton

$\hat{e}_1$: $-mg\sin(\theta) - F = ma$

$\hat{e}_2$: $N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$

While on Friction $0 \leq t \leq t_f$

$a = -g\sin(\theta) - \frac{1}{m}F$; but $F = \mu_k N = \mu_k mg\cos(\theta)$

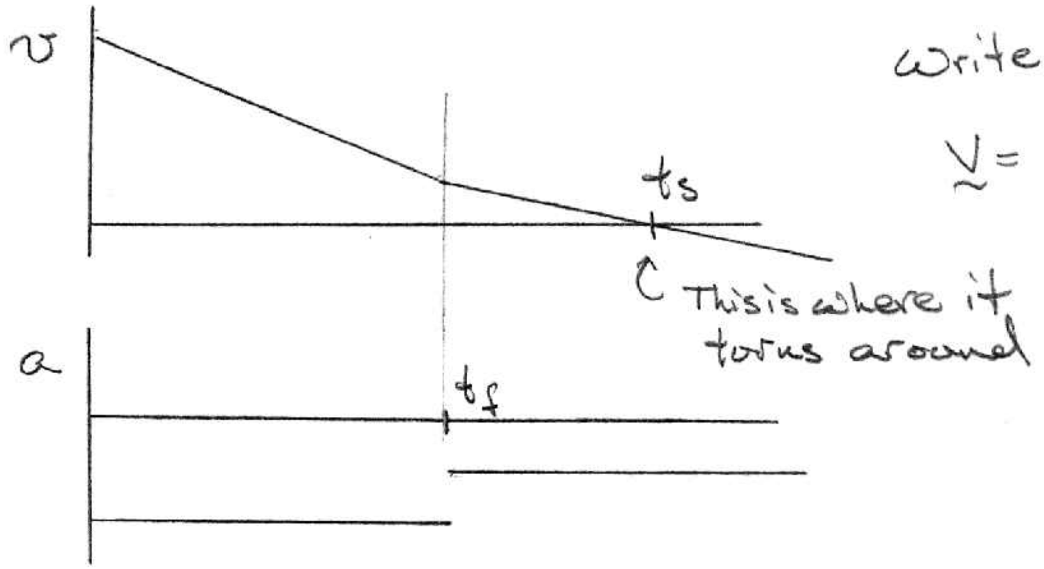
$\Rightarrow a = -g\sin(\theta) - \mu_k g\cos(\theta) = -g\left[\sin(\theta) + \mu_k\cos(\theta)\right]$

While not on friction $t_f \leq t < t_s$

$a = -g\sin(\theta)$

write velocity as

$\underset{\sim}{V} = v\,\hat{e}_1$



Kinematics while on friction part $0 \le t \le t_f$

$$v(t) = \int a\,dt = \int -g\left[\sin(\theta) + \mu_k \cos(\theta)\right] dt$$

$$v(t) = -g\left[\sin(\theta) + \mu_k \cos(\theta)\right] t + C_1$$

I.C.

$$v(0) = -g[\sim]\cdot 0 + C_1 = v_0 \Rightarrow C_1 = v_0$$

$$\Rightarrow v(t) = -g\left[\sin(\theta) + \mu_k \cos(\theta)\right] t + v_0$$

Note

$$v(t_f) = -g\left[\sin(\theta) + \mu_k \cos(\theta)\right] t_f + v_0$$

Kinematics on non-friction $t \ge t_f$

$$v(t) = \int -g\sin(\theta)\,dt = -g\sin(\theta)\,t + C_2$$

Boundary Condition

$$v(t_f) = -g\sin(\theta)\,t_f + C_2 = -g\left[\sin(\theta) + \mu_k \cos(\theta)\right] t_f + v_0$$

$$\Rightarrow \quad C_2 = -g\mu_k \cos(\theta)\, t_f + v_0$$

So

$$v(t) = -g\sin(\theta)\,t - \mu_k g \cos(\theta)\, t_f + v_0$$

Reaches highest point when speed is zero

$$v(t_s) = -g\sin(\theta)\,t_s - \mu_k g\cos(\theta)\,t_f + v_0 = 0$$

$$\Rightarrow \boxed{t_s = \frac{v_0 - \mu_k g \cos(\theta)\, t_f}{g\sin(\theta)}}$$

units $\quad \dfrac{L/T - L/T^2 \cdot T}{L/T^2} = \dfrac{L/T}{L/T^2} = T$ ✓