Chapter 2

Position, Velocity, and Acceleration

In this section, we will define the first set of quantities (position, velocity, and acceleration) that will form a foundation for most of what we do in this course.

2.1 Definition: Position

Position simply refers to the location of an object. For example, consider a bumble bee flying around a room. At any instant, you can mathematically define the location or position of the bee with a set of three numbers which we define as x, y, and z. We can define x as the distance of the bee from the south wall of the room; y as the distance of the bee from the east wall of the room; and z as the height or distance above the floor.

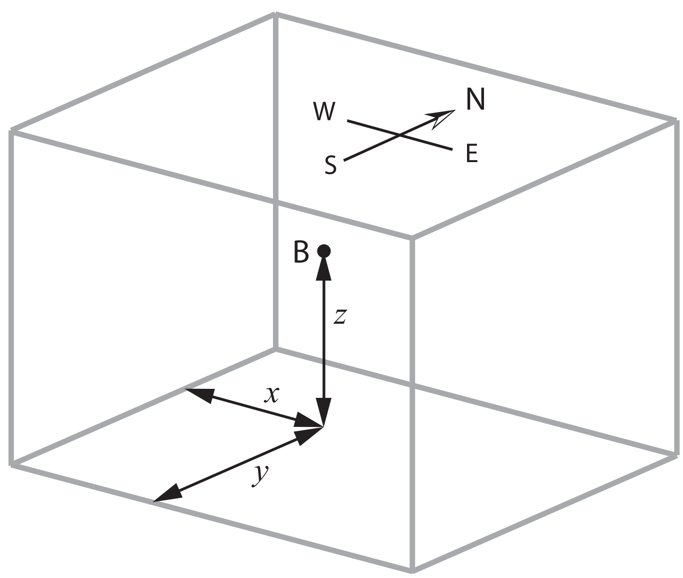

Figure 2.1: The position of a bee (point B) can be specified by three distances: x, y, and z.

2.1.1 Position as a Vector

Because we need three numbers to specify position, we can think of position as a vector:

$$\underset{\sim}{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}} + z\,\hat{\boldsymbol{k}}, \tag{2.1}$$

where x, y, and z are the components of position in the $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$ directions.

By now, you should be quite familiar with the concept of a vector. If it's fuzzy, I urge you to look at the Appendix on Review of Vectors. Graphically, a position vector looks like that shown in Figure 2.2. Recall that a vector is something that has both *magnitude and direction.* The position vector's *magnitude* (length of the vector) is

$$||\underset{\sim}{\boldsymbol{r}}|| = \sqrt{x^2 + y^2 + z^2}. \tag{2.2}$$

This is the distance of the bee from the origin of our coordinate system. The *direction* of the position vector is the direction of the bee as viewed from the origin of the coordinate system.

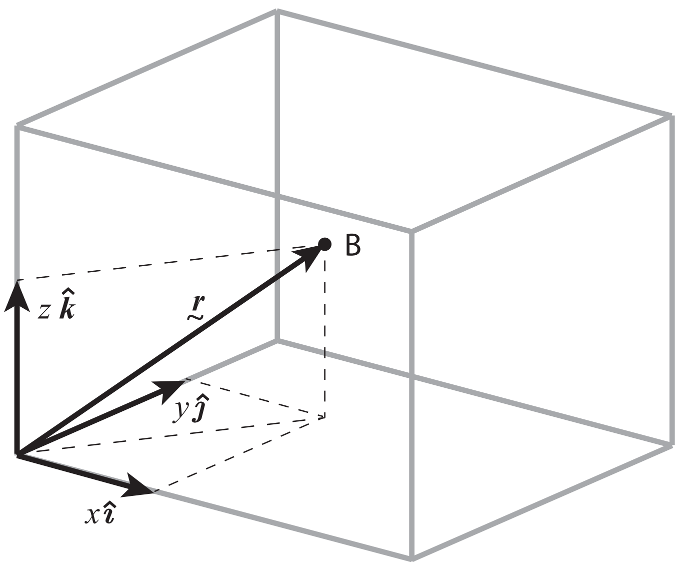

Figure 2.2: The position vector, $\underset{\sim}{\boldsymbol{r}}$.

2.1.2 Reduced Dimensions

Instead of being interested in a bee flying around a room, suppose that we are interested in a spider crawling on the floor. In this case, the spider's position can be written as

$$\underset{\sim}{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}} + 0\,\hat{\boldsymbol{k}}. \tag{2.3}$$

Here, the floor is defined as $z \equiv 0$, so the last term in the sum does not provide any important information and we can write

$$\underset{\sim}{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}}. \tag{2.4}$$

Here we say that the dynamics of the spider is two dimensional (2D), since is position can be described by only two coordinates: x, and y (assuming the spider doesn't begin climbing up any walls).

One dimensional (1D) dynamics are also possible. You can think of a train car traveling on a long straight segment of railroad track. In this case, one could write the velocity vector as

$$\underset{\sim}{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}}, \tag{2.5}$$

assuming the $\hat{\boldsymbol{\imath}}$ direction is in the direction of the track.

2.1.3 Position Has Units of Length

In mechanics all quantities have units of *mass*, *length*, *time*, or some combination of the three. Recall that, in our position vector of Equation (2.1), the symbols x, y, and z represented distances of the bee from different reference planes. Distances are *lengths*. Thus position has units of *length*. It is very important that you know this.

2.2 Definition: Velocity

I suspect that you already have some notion of what velocity is. As you're driving your car, you may look down at the speedometer and observe that you're traveling at a *rate* of 60 miles per hour. That 60 mph is your velocity, right?

Well, kind of. In this section we're going to carefully define what velocity is. In mechanics, we define velocity as follows: **Velocity is the time derivative of the position *vector*.**

Sounds simple, eh? If your understanding of time derivative is a bit fuzzy, I urge you to read the brief review in Appendix B.1. Our definition of velocity is a little trickier than the derivatives in the appendix because here we're taking the derivative of a vector.

As we're developing our understanding of velocity, I will refer to the picture in Figure 2.3(a). We can think of an ant walking along the path shown. At time $t = t_0$, we denote the position vector of the ant to be $\underset{\sim}{\boldsymbol{r}}(t_0)$.

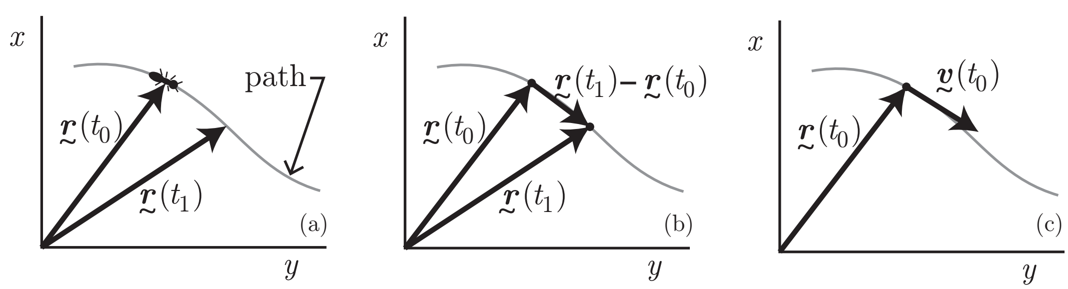

Figure 2.3: Definition of velocity.

The time derivative of the position *vector* at t_0 is given by

$$\underset{\sim}{\boldsymbol{v}}(t_0) = \left.\frac{d\underset{\sim}{\boldsymbol{r}}}{dt}\right|_{t_0} = \lim_{t_1 \to t_0} \frac{\underset{\sim}{\boldsymbol{r}}(t_1) - \underset{\sim}{\boldsymbol{r}}(t_0)}{t_1 - t_0}. \tag{2.6}$$

Because it is a derivative of a *vector*, it's a bit different than the time derivatives you saw in your first calculus class or in Appendix B.1. Nonetheless, the idea behind it is the same. Also, if you're having difficulty making sense of the vector subtraction in Figure 2.3, I urge you to look at Section A.3.2 in the Appendix.

Notice that the numerator is a difference between position vectors. Therefore, the numerator itself is a vector (shown in Figure 2.3). And when we divide the difference vector by the time difference, and take the limit, we get another vector, the velocity vector. The velocity vector is shown in Figure 2.3(c).

Here are some important observations to note about velocity:

- *Velocity is a vector.* According to the definition (2.6), velocity is a difference vector divided by a scalar. As a vector, the velocity has both magnitude and direction.

- The *direction* of the velocity vector is always *tangent to the path.* It points in the direction that the object is moving.

- *Units of velocity.* Recall that position has units of length (Section 2.1.3). Referring to Equation (2.6), we see that velocity is a length divided by time (*length per time*). When discussing velocity, we often put it in terms of miles per hour, meters per second. We can express velocity in units of inches per year if we wanted to.

- *Speed.* Velocity and speed are *not* the same thing. Velocity is a vector. Speed is a scalar; it is the magnitude of velocity. It is an instantaneous measure of how much distance is covered per unit of time, a measure of how fast the object is moving.

2.3 Definition: Acceleration

Acceleration is critically important in dynamics. To describe how forces hushing and pulling on an object affect the object's motion, we need to understand acceleration. We'll start with a definition: **Acceleration is the time derivative of the velocity *vector*.**

We can write the acceleration at time t_0 mathematically as

$$\underset{\sim}{\boldsymbol{a}}(t_0) = \left.\frac{d\underset{\sim}{\boldsymbol{v}}}{dt}\right|_{t_0} = \lim_{t_1 \to t_0} \frac{\underset{\sim}{\boldsymbol{v}}(t_1) - \underset{\sim}{\boldsymbol{v}}(t_0)}{t_1 - t_0}. \tag{2.7}$$

Since velocity is a the derivative of position with respect to time, we can write acceleration as a second derivative:

$$\underset{\sim}{\boldsymbol{a}}(t_0) = \left.\frac{d^2\underset{\sim}{\boldsymbol{r}}}{dt^2}\right|_{t_0}$$

2.3.1 Acceleration Example # 1

To help us think about what acceleration means, I want us to work through a couple examples. The first is depicted in Figure 2.4, in which we imagine a car driving on a straight, flat road. The

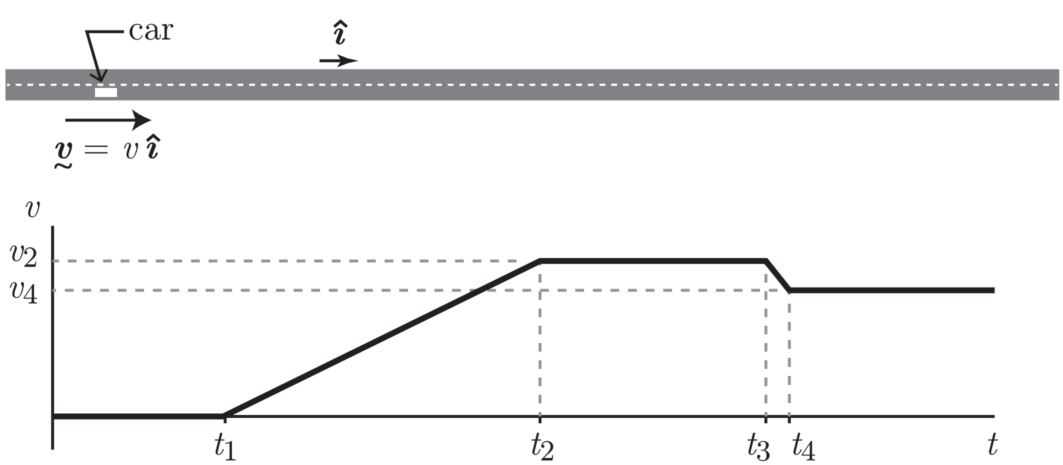

Figure 2.4: Top view of a car driving on a straight flat road.

figure also shows a plot of the car's *speed* v as a function of time. The car's *velocity* vector is given by

$$\underset{\sim}{\boldsymbol{v}} = v\,\hat{\boldsymbol{\imath}}. \tag{2.8}$$

Initially, the car is stopped at a stop sign. It is motionless. Then, at time t_1, the car's speed begins increasing linearly. At time t_2, the car reaches and then temporarily maintains a constant speed v_2. At time t_3, suppose that the driver notices a police cruiser and immediately begins slowing down. At time t_4, the car reaches the speed limit v_4, and then maintains that speed.

The objective of this exercise is to find the acceleration of the car as a function of time.

Using (2.8) and our definition of acceleration (2.7), we get

$$\underset{\sim}{\boldsymbol{a}} = \dot{\underset{\sim}{\boldsymbol{v}}} = \dot{v}\,\hat{\boldsymbol{\imath}}. \tag{2.9}$$

Thus, acceleration can be obtained by simply taking the time derivative of the speed curve in Figure 2.4. Our plot of $a = \dot{v}$ as a function of time is shown in Figure 2.5.

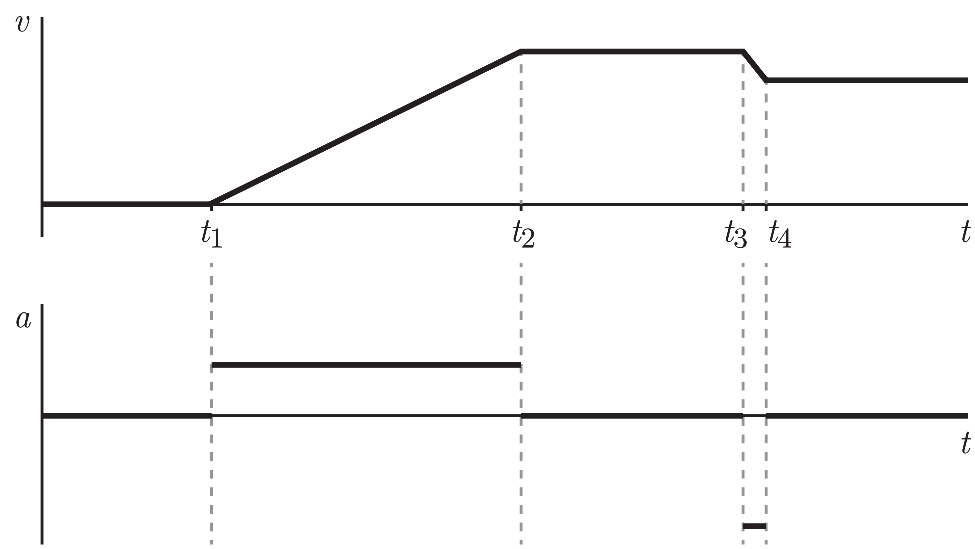

Figure 2.5: Plots of the $\boldsymbol{i}$ components of velocity and acceleration.

We should be careful in how we discuss these quantities. It is a *mistake* to say that the a in the plot is acceleration. Why? Because acceleration is a *vector*. The acceleration vector is $\underset{\sim}{\boldsymbol{a}} = a\,\hat{\boldsymbol{\imath}}$. So the scalar a is the $\hat{\boldsymbol{\imath}}$ component (only non-zero component) of acceleration[1].

Note that for $t < t_1$, the car is motionless and there is no acceleration. From t_1 through t_2, the velocity in the $\hat{\boldsymbol{\imath}}$ direction is increasing. Therefore, we have acceleration in the positive $\hat{\boldsymbol{\imath}}$ direction. Since speed is increasing at a constant rate (constant slope), the acceleration is constant between t_1 and t_2.

The car is cruising its maximum speed between t_2 and t_3. Since that speed is *constant* (unchanging) in this time interval, the acceleration is *zero.* Similarly, for $t > t_4$, acceleration is zero when the car is traveling at a different constant rate, at the speed limit.

Between t_3 and t_4, the speed of the car is decreasing. The slope of the v versus t curve is negative. Therefore, the acceleration is in the negative $\hat{\boldsymbol{\imath}}$ direction.

[1] If my discussion about vectors is confusing I urge you to read through Appendix A.

Note that the slope of the v versus t is steeper (negative) between t_3 and t_4 than it is between t_1 and t_2. As a result, the acceleration between t_3 and t_4 has a bigger magnitude than the acceleration between t_1 and t_2. If there are aspects of taking derivatives that seem fuzzy to you, I urge you to read Appendix B.1.

A Geometric Perspective

In calculating the acceleration above, we used Equation (2.9) and then took the derivative of the $\boldsymbol{i}$ component of velocity (a scalar). It might be instructive to try taking a derivative of the velocity vector directly.

Recall from (2.7), that the acceleration vector is a ratio. Change in the velocity vector is in the numerator; change in time is in the denominator. In Figure 2.6, I show the plot of the car's speed again, along with four pairs of times at at which we can examine the change in velocity. Each of the pairs of times are spaced a time Δt apart as indicated in the figure.

We'll start by looking at part (w) of Figure 2.6. On the left side, I show the velocity vector at times t_A and t_B. I also show the difference $\boldsymbol{v}(t_b) - \boldsymbol{v}(t_a)$. On the right side of Figure 2.6, I show the vector

$$\frac{\boldsymbol{v}(t_b) - \boldsymbol{v}(t_a)}{\Delta t}, \tag{2.10}$$

which, except for the limit part, is the acceleration vector[2]. Because the difference vector $\boldsymbol{v}(t_b) - \boldsymbol{v}(t_a)$ is in the positive $\boldsymbol{i}$ direction, the acceleration vector is also in the positive $\boldsymbol{i}$ direction.

In Figure 2.6(x), we do the same thing at times t_c and t_d. Although the velocity vectors $\boldsymbol{v}(t_c)$ and $\boldsymbol{v}(t_d)$ are "bigger" than $\boldsymbol{v}(t_a)$ and $\boldsymbol{v}(t_b)$, the differences $\boldsymbol{v}(t_d) - \boldsymbol{v}(t_c)$ and $\boldsymbol{v}(t_b) - \boldsymbol{v}(t_a)$ are the same. This is because the Δts are the same in both cases and the slopes of $v(t)$ are the same. Therefore the accelerations are the same. In fact, we get the same acceleration vector in the positive $\boldsymbol{i}$ direction in the entire interval between t_1 and t_2.

At times t_e and t_f, the velocities are the same, and thus the difference $\boldsymbol{v}(t_f) - \boldsymbol{v}(t_e)$ is $\boldsymbol{0}$, the zero vector. The numerator is the zero vector, and thus the acceleration is the zero vector.

When we repeat the process at t_g and t_h, we find that the difference vector $\boldsymbol{v}(t_h) - \boldsymbol{v}(t_g)$ points in the negative $\boldsymbol{i}$ direction. It's because the velocity is getting "smaller". Thus, the acceleration vector is in the negative $\boldsymbol{i}$ direction. Because the difference $\boldsymbol{v}(t_h) - \boldsymbol{v}(t_g)$ has bigger magnitude than that of the difference $\boldsymbol{v}(t_b) - \boldsymbol{v}(t_a)$, and the two Δts are the same, the negative acceleration in Figure 2.6(z) has bigger magnitude than the positive acceleration in Figure 2.6(w).

2.3.2 Acceleration Example # 2

In this next example, I want you to think of a car driving at a constant speed of 55 mph on a flat but winding road. As shown in Figure 2.7, the initial segment of road, up to point A, is straight. From point A to point B, the car makes a gentle left turn along a circular arc of radius R_{AB}. Between points B and C, the track is straight again. Then the car makes a right turn on a circular arc from point C to point D. The radius of the second arc, R_{CD} is smaller than the radius of the first arc R_{AB}. Therefore, the second turn is a bit "sharper" than the first. After point D, the road is straight again.

Again, the objective of the exercise is to ***find the acceleration of the car as it travels along the path shown.***

[2] It's rather straightforward to show that, in this case, the acceleration vector and (2.10), without the limit, are identical.

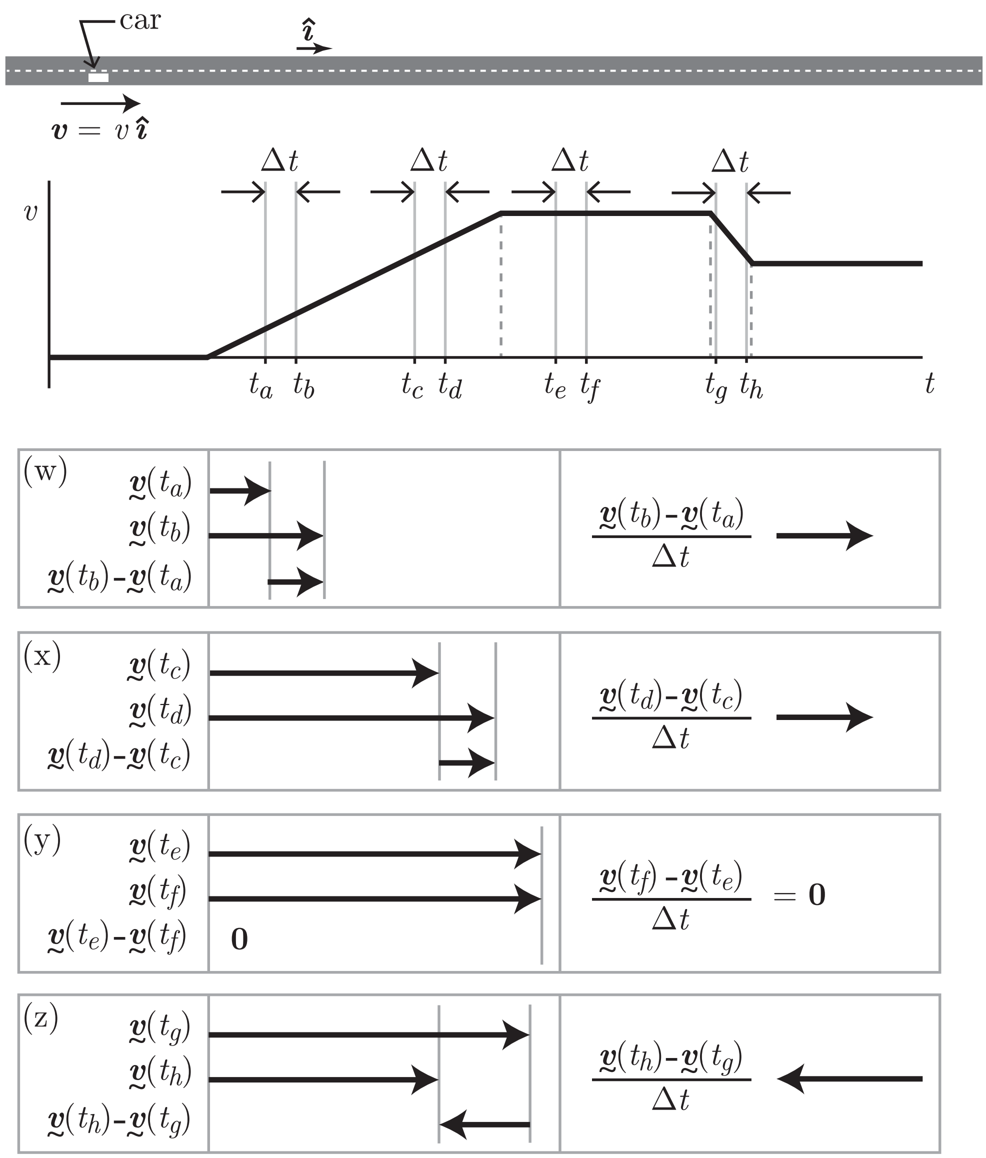

Figure 2.6: A geometric perspective for looking at acceleration.

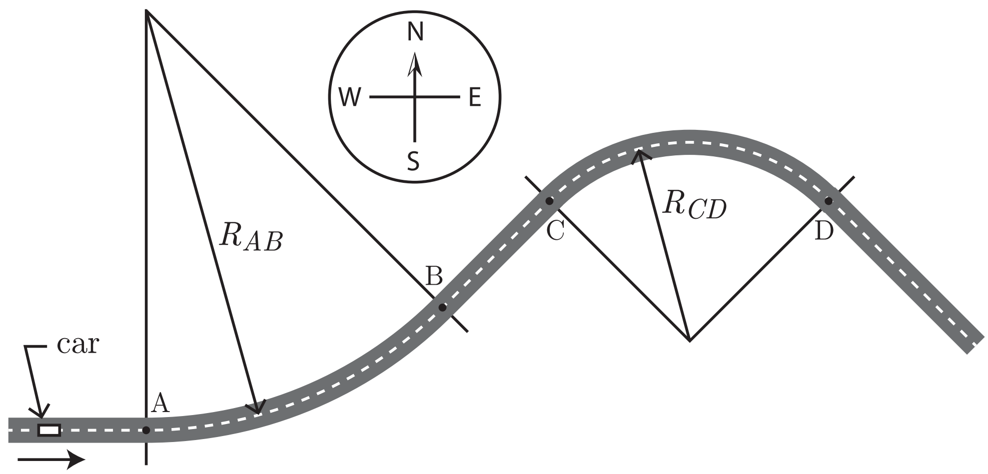

Figure 2.7: A car that drives at a constant speed on a winding path (top view).

A trick question?

At first glance, this may appear to be a trick question. In the first sentence of the section, I say that the car is traveling at a *constant* speed. Well, if the speed is constant, then the time derivative of the speed is zero. Therefore, the acceleration must be zero, right?

Be careful! Acceleration is *not* the derivative of speed. It is the derivative of the *velocity vector*. Since the speed is constant, the magnitude of the velocity vector does not change. However, the direction of the velocity vector changes, doesn't it?

At point A, the car is traveling due East (according to the compass directions labeled in Figure 2.7). At points B and C, the car is traveling northeast. At point D, the car is traveling southeast.

Recall from Section 2.2 that the velocity vector is always tangent to the path, pointing in the direction of the car's motion. As the car travels along the curvy path of Figure 2.7, the velocity vector changes because its direction changes. Therefore the acceleration vector, generally, is not zero.

Acceleration Due to Path Curvature

In Figure 2.8, I show the path (dashed) of the car through the first left turn, and I label two points on the path "a" and "b." We let t_a and t_b denote the times at which the car passes points "a" and "b." Furthermore, I've drawn the velocity vectors at these two points: $\underset{\sim}{\boldsymbol{v}}(t_a)$ and $\underset{\sim}{\boldsymbol{v}}(t_b)$.

Note that the two velocity vectors have the same magnitude (length), because speed is constant – it's the same everywhere. However, in terms of direction, we see that velocity is always tangent to the path. as the car moves along the path, the direction of the velocity vector changes.

At the lower right side of Figure 2.8, I enlarge the velocity vectors a bit, just so that it's easier to see all the pieces. When we place the two velocity vectors together tail-to-tail, it becomes easy to visualize the difference $\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)$. Then, when we take the velocity difference and divide by the time difference $t_b - t_a$, we get a vector approximating the acceleration, shown in the upper right corner of the figure.

Aha! We see that the acceleration vector appears to point toward the inside part of the curve. Note that our approximation of the acceleration vector is almost perpendicular to the velocity vector.

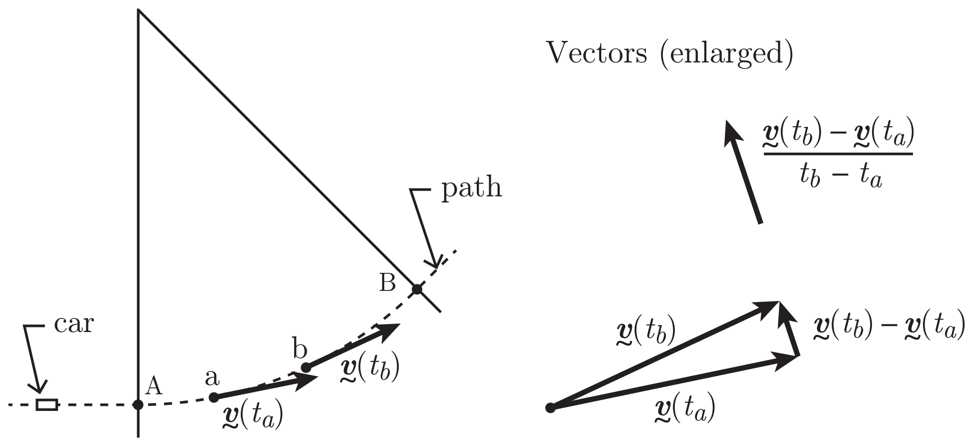

Figure 2.8: Velocity vectors and differences in the first left turn of the road.

A Closer Look at the Direction of Acceleration

To examine the direction of the acceleration vector more closely, let's consider the vector triangle containing $\underset{\sim}{\boldsymbol{v}}(t_a)$, $\underset{\sim}{\boldsymbol{v}}(t_b)$, and $\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)$. I reproduce the triangle in Figure 2.9, even larger, so that we can see the angles better.

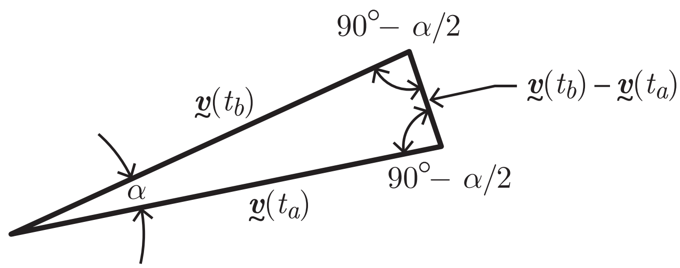

Figure 2.9: Velocity triangle from Figure 2.8.

Here, we let α denote the angle between the two velocity vectors. And since the two velocity vectors have the same magnitude, the two long sides of the triangle have the same length. The triangle is an isosceles triangle. As a consequence, the two other interior angles must be $90^\circ - \alpha/2$.

Therefore, our approximation of the acceleration isn't quite perpendicular to the velocity vector. The angle between the two is $90^\circ - \alpha/2$ rather than 90°. But it's almost perpendicular since α is small.

Now recall that the actual acceleration, since it is a derivative, has a limit in it:

$$\underset{\sim}{\boldsymbol{a}}(t_a) = \lim_{t_b \to t_a} \frac{\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)}{t_b - t_a}. \tag{2.11}$$

So when we let t_b approach t_a in the limit, the two vectors get closer together; the angle α gets smaller. Thus, in the limit, the *acceleration vector **is** perpendicular to the velocity vector.* And since the velocity vector is tangent to the path, we can say the acceleration vector is perpendicular to the path.

Thinking About Assumptions

At the beginning of Section 2.3.2, we said that the road between points A and B was a circular path. However, none of the arguments we've made thus far required the path to be circular. The path just needed to be curved.

Our results so far simply state that when an object is moving at a constant speed along a curved path, its acceleration is perpendicular to the path and points toward the "inside" of the curve.

Dependence on Speed

At the beginning of Section 2.3.2, I stated that the car was traveling at a constant speed of 55 mph. How does the acceleration differ if the car was going at a constant speed of 110 mph instead?

Let's redraw Figure 2.8, but this time we'll do it for the faster speed. The result is in Figure 2.10. For reference, I have shown the previous vectors from Figure 2.8 in light gray.

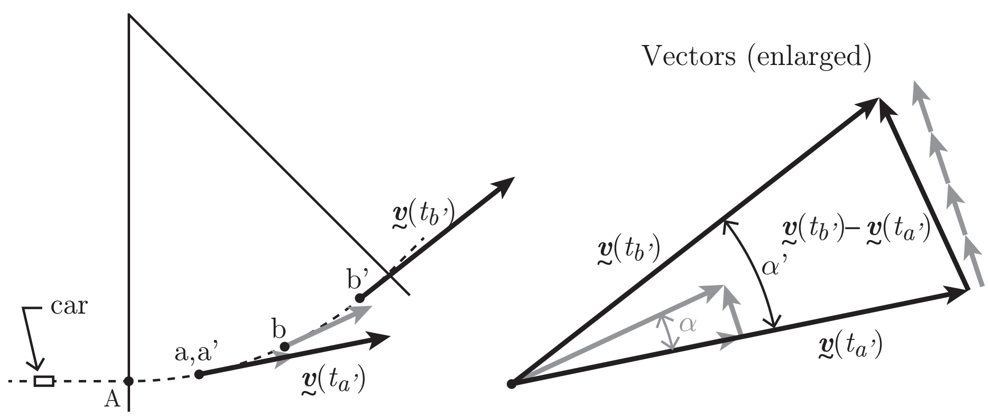

Figure 2.10: Velocity vector triangle analogous to that of Figure 2.8, except the car is traveling twice as fast. Previous results are shown in light gray.

Recall that when the car was traveling at 55 mph, we defined two points labeled "a" and "b" in Figure 2.8. Similarly, we define points a' and b' in Figure 2.10. The point a' is chosen so that it coincides with point "a" exactly. Then, point b' is chosen so that the time differences are the same,

$$t_{b'} - t_{a'} = t_b - t_a. \tag{2.12}$$

Point b' is twice as far down the track than point "b" because the car is going twice as fast: it covers twice the distance in the same amount of time.

The new velocity vector triangle is shown on the right half of Figure 2.10. Notice that the vectors $\underset{\sim}{\boldsymbol{v}}(t_{a'})$ and $\underset{\sim}{\boldsymbol{v}}(t_{b'})$ are twice as large as the vectors $\underset{\sim}{\boldsymbol{v}}(t_a)$ and $\underset{\sim}{\boldsymbol{v}}(t_b)$.

If doubling the length of two sides of the velocity triangle were the only effects of doubling the speed, then the difference $\underset{\sim}{\boldsymbol{v}}(t_{b'}) - \underset{\sim}{\boldsymbol{v}}(t_{a'})$ would double in size as well. But here, we also double the size of the interior angle $\alpha' = 2\alpha$. [Recall that point b' is twice as far down the circular track.] This causes the difference to double again. As a result, $\underset{\sim}{\boldsymbol{v}}(t_{b'}) - \underset{\sim}{\boldsymbol{v}}(t_{a'})$ is about four times larger than the original difference $\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)$.

Recall that the time differences (2.12) are identical. Therefore the acceleration for the 110 mph case predicted by Figure 2.10 is approximately four times larger in magnitude than the acceleration predicted in the 55 mph case of Figure 2.8. The fact that doubling the speed causes the amplitude to quadruple in magnitude suggests that the acceleration magnitude is proportional to speed squared:

$$|\underset{\sim}{\boldsymbol{a}}| \sim v^2. \tag{2.13}$$

Dependence on Radius of Curvature

You may recall in Figure 2.7 that the first curve is rather gentle while the second is sharper. We can quantify the "sharpness" of the curve by the radius of the circular arc. The smaller the radius, the sharper the curve. Now we can ask the question: ***How does the acceleration depend on the radius of curvature?***

In Figure 2.11, we show the gentle left turn as well as the sharper right turn. The radius of the right turn is half as large as the radius for the left turn: $R_{AB} = 2\,R_{CD}$. We'll assume that the car is traveling at a constant rate of 55 mph as we did originally.

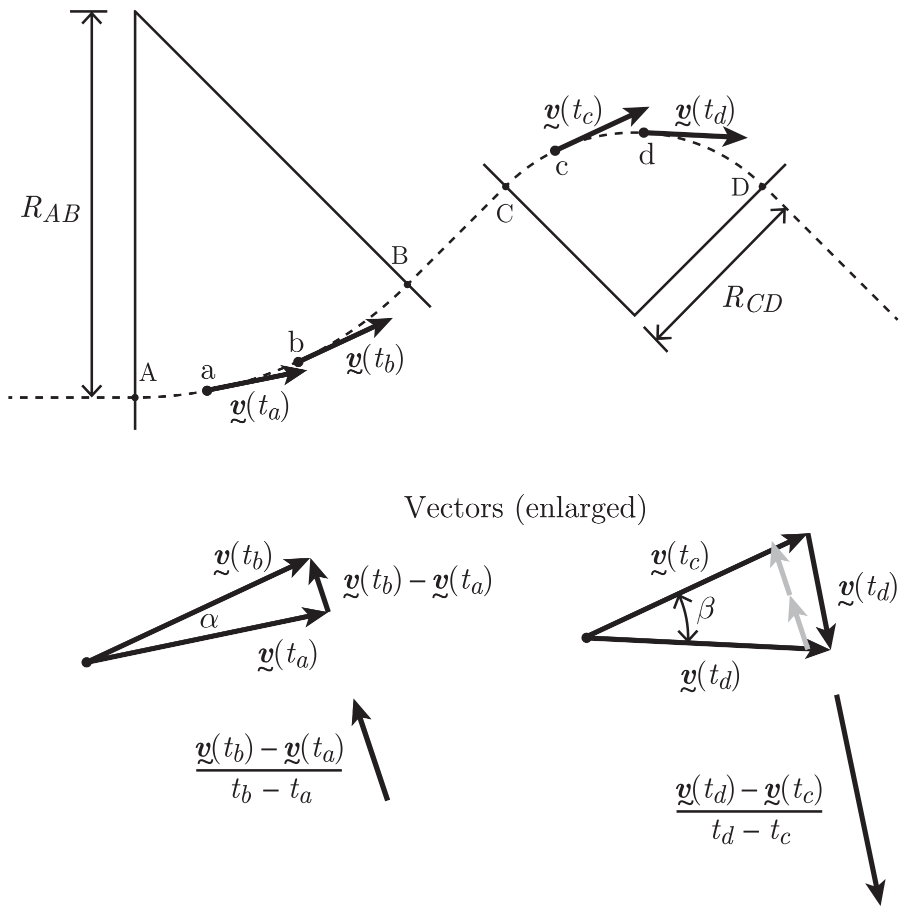

Figure 2.11: Velocity vector triangles for the gentle left turn and the sharper right turn.

The velocity vectors for the gentle left turn are the same as those already presented in Figure 2.7. The new information lies on the right side of the figure, where I defined points "c" and "d" so that the distance between points "c" and "d" (along the circular arc) is the same as the distance between points "a" and "b". Because the car is traveling at the same speed along the entire track, we get $t_d - t_c = t_b - t_a$.

Observe that the velocity vectors $\underset{\sim}{\boldsymbol{v}}(t_c)$ and $\underset{\sim}{\boldsymbol{v}}(t_d)$ have the same magnitude, and they have the same magnitude as $\underset{\sim}{\boldsymbol{v}}(t_a)$ and $\underset{\sim}{\boldsymbol{v}}(t_b)$. Notice, however, that the direction of the car changes more rapidly in the sharper turn. The interior angle β for the right turn is twice as big as the interior angle for the left turn, α. As a consequence, the velocity difference for the sharp turn, $\underset{\sim}{\boldsymbol{v}}(t_d) - \underset{\sim}{\boldsymbol{v}}(t_c)$, is roughly twice as big in magnitude as the velocity difference for the gentle turn, $\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)$. Therefore, our estimates of acceleration come in roughly a two-to-one ratio:

$$\frac{\underset{\sim}{\boldsymbol{v}}(t_d) - \underset{\sim}{\boldsymbol{v}}(t_c)}{t_d - t_c} \approx 2\,\frac{\underset{\sim}{\boldsymbol{v}}(t_b) - \underset{\sim}{\boldsymbol{v}}(t_a)}{t_b - t_a}.$$

As we'll show in Section ??, the approximation above becomes exact when we take the limit and calculate the derivative properly.

Combining this result with the dependence on acceleration expressed in Equation (2.13), we get

$$|\underset{\sim}{\boldsymbol{a}}| \sim \frac{v^2}{R}. \tag{2.14}$$

Acceleration on the Road as a Whole

In Figure 2.12, I show acceleration vectors — magnitude and direction — at several points along the road. The points are roughly equally spaced.

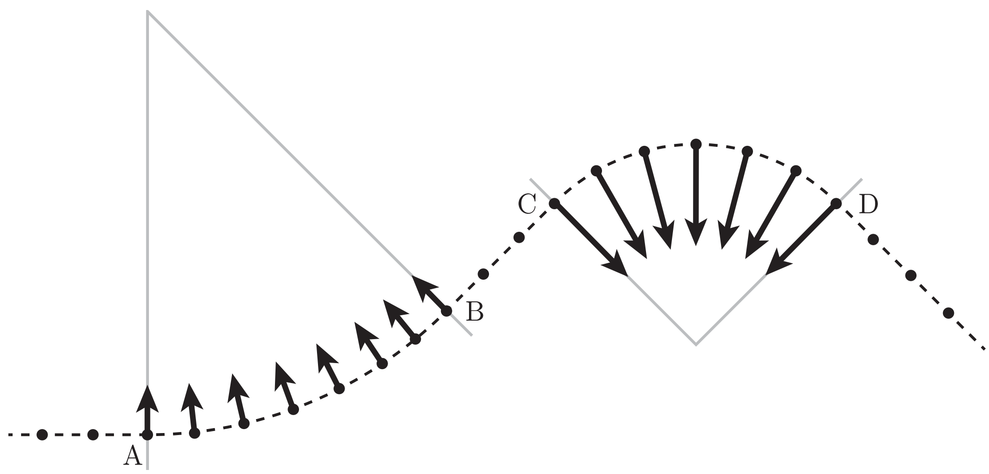

Figure 2.12: Acceleration vectors at several points along the path.

The first thing to note is that, in the straight sections of the path, there is no acceleration. The velocity vector is not changing its magnitude or direction.

In the curved portions of the track, the acceleration vectors are perpendicular to the path and point toward the "center of curvature." Because the right turn is a "sharper" turn, the acceleration vectors there are larger in magnitude.

Something to think about. In this exercise we assumed that the car was traveling along the track from left to right at a constant speed of 55 mph. Now suppose that the car is traveling from right to left at 55 mph. How does the acceleration change under these circumstances?

2.3.3 Acceleration, Combining Examples #1 and #2

You'll recall, that in Example 1, the road was straight, but the speed changed. Then, in Example #2, we held the speed constant, but allowed the direction to change. In the first case, the acceleration was pointing in the direction of the path. In the second case, the acceleration was perpendicular to the path.

When the car is able to execute a more general motion in which it is allowed to change its direction *and* change its speed, it has both types of acceleration. We can write the acceleration as

$$\underset{\sim}{a} = a_t\,\hat{e}_t + a_n\,\hat{e}_n. \tag{2.15}$$

Here, $\boldsymbol{e}_t$ and $\boldsymbol{e}_n$ are basis vectors that point tangent and perpendicular (normal) to the path respectively. In Figure 2.13, I draw examples of $\hat{e}_t$ and $\hat{e}_n$ at different points along the path.

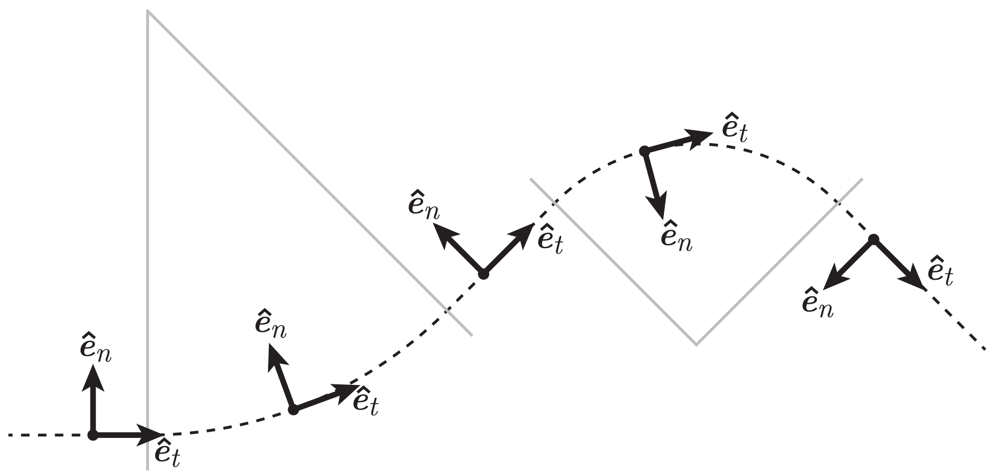

Figure 2.13: Basis vectors for tangential and perpendicular (normal) directions.

The *tangengential component* of acceleration is simply the time derivative of the speed as we found in Section 2.3.1:

$$a_t = \dot{v}. \tag{2.16}$$

The *normal component* of accelerate is proportional to speed squared divided by radius of curvature.

$$a_n \sim \frac{v^2}{R}. \tag{2.17}$$

In Chapter ???, we will derive the expressions for a_t and a_n more rigorously.

2.3.4 Units of Acceleration

With position and velocity, we were careful to specify unit. It's a theme we'll continue throughout the semester. We can deduce the units of acceleration by examining the definition in Equation (2.7). Notice that the numerator on the right hand side is a velocity, which has units of length per time. Meanwhile, the denominator has units of time. Therefore, *acceleration has units of length per time squared.* (L/T^2)

Now in the previous section, we said that the normal component of acceleration is proportional to v^2/R. Let's look at the units of v^2/R. Recall that speed, v, has units of length per time (L/T), while R is the radius, a length (L). Therefore,

$$\left[\frac{v^2}{R}\right] = \frac{\left(\frac{L}{T}\right)^2}{L} = \frac{L^2}{T^2\,L} = \frac{L}{T^2}. \tag{2.18}$$

Aha! The quantity v^2/R is a length per time squared, just like acceleration. This is no coincidence.

2.4 Take-Aways

Below is a list of what I feel to be the most important points that I want you to "take away" from Chapter 2.

1. Position is a vector. It has both magnitude *and* direction.
2. Position has units of *length.* (L)
3. Velocity is a vector. It is the time derivative of the position vector.
   (a) The *direction* of the velocity vector points in the direction that the object is moving, tangent to the path.
   (b) The magnitude of the velocity vector is a measure of how fast the object is moving.
4. Velocity has units of *length per time.* (L/T)
5. *Speed* is the magnitude of the velocity vector. It is a scalar.
6. Acceleration is a vector. It is the time derivative of the velocity vector. It is the second time derivative of the position vector.
   (a) The component *tangent* to the path is due to the changing speed of the object. The component magnitude is equal to $\dot{v}$.
   (b) The component *normal* (perpendicular) to the path is due to the changing direction of the object's velocity. This component points toward the center of curvature and has magnitude that is proportional to v^2/R, where R is the radius of curvature.
7. Acceleration has units of *length per time squared.* (L/T^2).