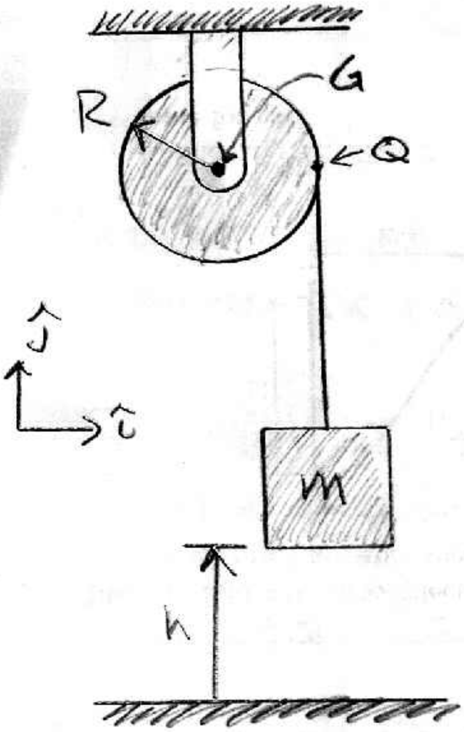
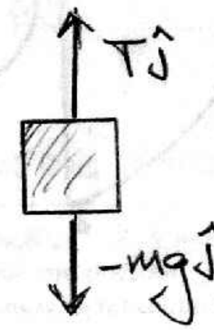


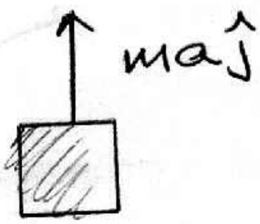
Given: m, m_p, I_G, h, R

Find: $t_G \leftarrow$ Time required to hit ground

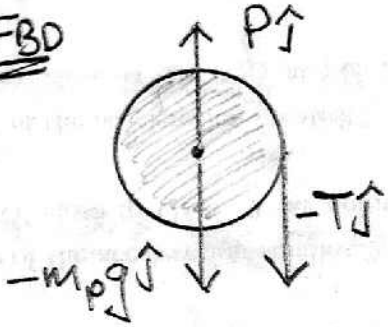
FBD



MAD



FBD



MAD

Newton (Block)

$$\hat{j}: \; T - mg = ma \qquad (1)$$

Euler (Spool): $\Sigma \underset{\sim}{M}_G = I_G \underset{\sim}{\alpha}$

$$\hat{k}: \; -TR = I_G\alpha \qquad (2)$$

Substitute (1) into (2)

$$-R(mg + ma) = I_G\alpha$$

or

$$-Rmg = I_G\alpha + mRa \qquad (3)$$

In (3), there are two unknowns: $\underbrace{a \,\&\, \alpha}$.

They can be related kinematically

Consider point Q, where the rope comes in contact with the spool

The point Q on the rope has the same acceleration as the block

$$\underset{\sim}{a}_Q^{(\text{rope})} = a\hat{j}$$

The point Q on the spool is

$$\underset{\sim}{a}_Q^{(\text{spool})} = \cancel{\underset{\sim}{a}_G}^{\,0} + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{Q/G} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{Q/G})$$

$$= R\alpha\hat{j} - R\omega^2\hat{i}$$

The vertical components of these accelerations must match

$$\Rightarrow a = R\alpha$$

Subst into (3)

$$-Rmg = \frac{I_G}{R} a + mRa$$

$$\Rightarrow a = \frac{-g}{\left(1 + \frac{I_G}{mR^2}\right)} \quad (4)$$

Observe that a is constant

Let $\underset{\sim}{v} = v\hat{j}$ & $\underset{\sim}{r} = y\hat{j}$ denote velocity & position of the block

$$v = \int a\,dt = at + c_1 \; ; \quad \text{I.C. } v(0) = a\cdot 0 + c_1 = 0$$
$$\Rightarrow c_1 = 0$$
$$\Rightarrow v(t) = at$$

$$y(t) = \int v(t)\,dt = \int at\,dt = \tfrac{1}{2}at^2 + c_2$$
$$\text{I.C. } y(0) = \tfrac{1}{2}a\cdot 0^2 + c_2 = h \Rightarrow c_2 = h$$
$$y(t) = \tfrac{1}{2}at^2 + h$$

Let t_G be time at which box hits ground

$$y(t_G) = \tfrac{1}{2}at_G^2 + h = 0 \quad \Rightarrow t_G = \sqrt{\frac{-2h}{a}}$$

Subst (4) into above

$$\boxed{t_G = \sqrt{\frac{2h}{g}\left(1 + \frac{I_G}{mR^2}\right)}}$$

units $\sqrt{\frac{L}{L/T^2}\left(1 + \frac{mL^2}{mL^2}\right)} = \sqrt{T^2} = T$ ✓

If we double I_G : $I_G^{(new)} = 2I_G$

original moment of inertia

$$\boxed{t_G^{(new)} = \sqrt{\frac{2h}{g}\left(1 + \frac{2I_G}{mR^2}\right)}}$$

takes more time to hit ground