**Given:** $M, M_p, I_g, h, R$

**Find:** $t_g \leq$ Time required to hit ground

\[ FBD \]

\[ \sum F_j = m a \]  

\[ Euler (Spool): \sum M_\alpha = I_g \alpha \]

\[ K: -TR = I_g \alpha \]  

Substitute (1) into (2)

\[ -R(mg + ma) = I_g \alpha \]

or

\[ -Rmg = I_g \alpha + mRa \]  

\[ \text{(3)} \]
In (3), there are two unknowns: $a$ and $\omega$. They can be related kinematically.

Consider point $Q$, where the rope comes in contact with the spool.

The point $Q$ on the rope has the same acceleration as the block:

$$a_{(\text{rope})} = a \hat{j}$$

The point $Q$ on the spool is

$$a_{(\text{spool})} = \omega \times \omega \times \frac{I}{2} + \omega \times (\omega \times \omega \times \frac{I}{2})$$

$$= R \omega \hat{j} - RW^2 \hat{i}$$

The vertical components of these accelerations must match:

$$\Rightarrow a = R \omega$$

Subst. into (3)

$$-Rmg = \frac{I}{R} a + Ma$$

$$\Rightarrow a = \frac{-g}{\left(1 + \frac{I}{MR^2}\right)}$$ (4)
Observe that $a$ is constant

Let $\mathbf{v} = v \mathbf{j}$ & $\mathbf{r} = r \mathbf{j}$ denote velocity & position of the block.

$\mathbf{v} = \int a \, dt = a + c_1$ ; I.e. $\mathbf{v}(0) = a \cdot 0 + c_1 = 0 \implies c_1 = 0$

$\implies \mathbf{v}(t) = at$

$\mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int at \, dt = \frac{1}{2}at^2 + c_2$

I.e. $\mathbf{r}(t) = \frac{1}{2}at^2 + c_2 = h \implies c_2 = h$

$\mathbf{r}(t) = \frac{1}{2}at^2 + h$

Let $t_4$ be the time at which box hits ground.

$\mathbf{r}(t_4) = \frac{1}{2}at_4^2 + h = 0 \implies t_4 = \sqrt{\frac{-2h}{a}}$

Subsitute (4) into above

$$t_4 = \sqrt{\frac{2h}{g} \left(1 + \frac{I_4}{I_4 \cdot m R^2}\right)}$$

Units

$$\sqrt{\frac{L}{I R^2} \left(1 + \frac{m L^2}{m L^2}\right)} = \sqrt{T^2} = T \checkmark$$
If we double $I_G$: $I_G^{(new)} = 2I_G$

$\varepsilon^{(new)} = \sqrt{\frac{2h}{g} \left(1 + \frac{2I_G}{mR^2}\right)}$

... takes more time to hit ground...