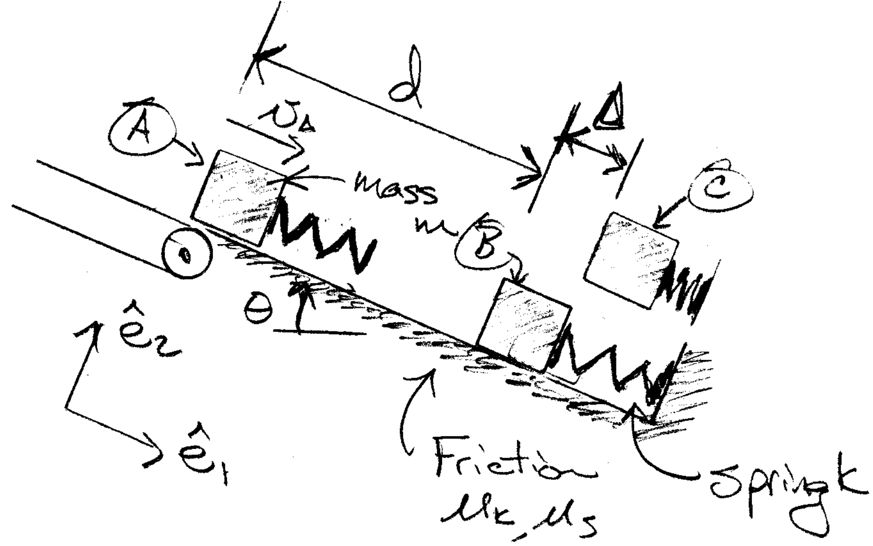
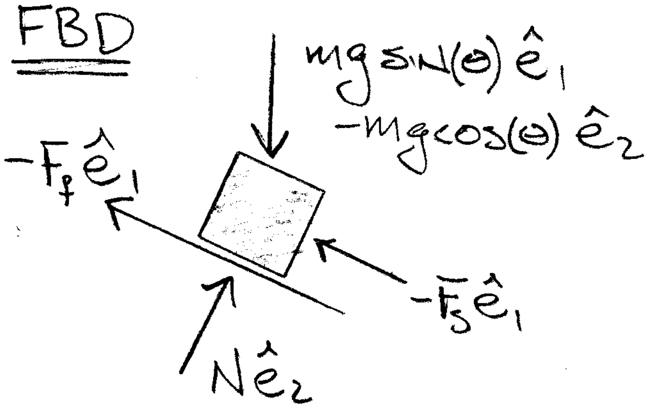


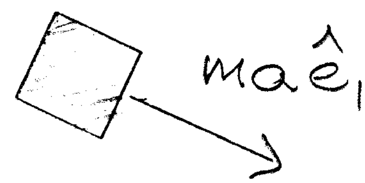

Given: $m, \mu_k, \theta, K,$
$d, F_{max}$

Find: Maximum speed $v_A$ for which impact force does not exceed $F_{max}$

FBD




MAD




Newton $\hat{e}_2$: $N - mg\cos(\theta) = 0$ ← No accel in $\hat{e}_2$

$$\Rightarrow N = mg\cos(\theta)$$

So, while block is sliding to the right, friction is to the left

$$F_f = \mu_k N = \mu_k mg\cos(\theta)$$

Also, the spring force: $F_s = K\delta$ ← Amount the spring is compressed

Note: The "impact" force is that of the spring. The spring force is largest at maximum compression

$\Rightarrow \quad F_{max} = K\Delta$ ← max spring compression

↑ max spring force

$\Rightarrow \Delta = \frac{F_{max}}{K} \quad (1)$

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<u>Work Energy</u>: $U_{A \to C} = \cancel{T_C}^{\,0} - T_A \quad (2)$

- Let state (A) be when block is at top of ramp.
- Let state (C) be when spring is at maximum compression
  $\Rightarrow v_C = 0 \Rightarrow T_C = 0$

Work by gravity:

$$U^{(grav)}_{A \to C} = mg\,(d+\Delta)\sin(\theta)$$

Work by spring:

$$U^{(spring)}_{A \to C} = -\frac{1}{2} K \Delta^2$$

Work by constant friction force

$$U^{(frict)}_{A \to C} = -\mu_k\, mg \cos(\theta)\,(d+\Delta)$$

Put all the pieces into (2)

$$mg(d+\Delta)\sin(\theta) - \frac{1}{2}K\Delta^2 - mg(d+\Delta)\mu_k\cos(\theta) = -\frac{1}{2}v_A^2$$

$$\Rightarrow v_A = \sqrt{\frac{K}{m}\Delta^2 - 2g\left[\sin(\theta) - \mu_k\cos(\theta)\right](d+\Delta)}$$

Substitute (1)

$$\boxed{v_A = \sqrt{\frac{F_{max}^2}{mK} - 2g\left[\sin(\theta) - \mu_k\cos(\theta)\right]\left(d + \frac{F_{max}}{K}\right)}}$$

Unit Check

$$\sqrt{\frac{\left(\frac{ML}{T^2}\right)^2}{M\,\frac{M}{T^2}} - \frac{L}{T^2}\left(L + \frac{\frac{ML}{T^2}}{\frac{M}{T^2}}\right)}$$

$$= \sqrt{\frac{L^2}{T^2} - \frac{L}{T^2}(L+L)} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} \checkmark$$