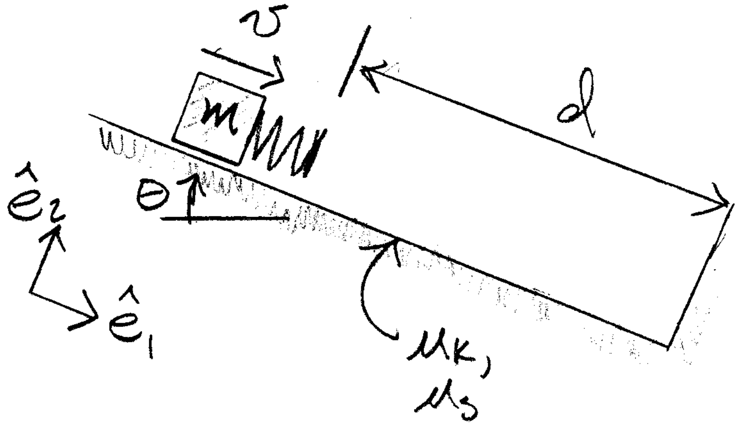
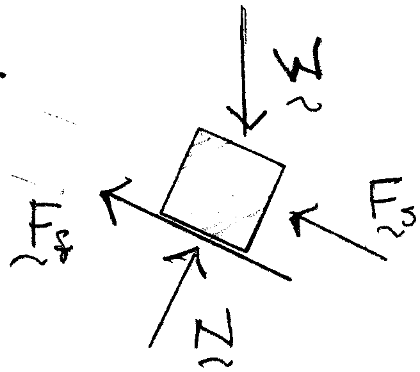


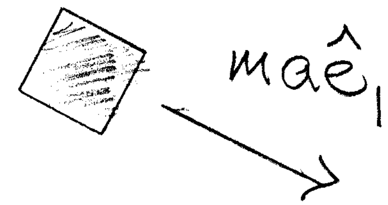
Given: m, d, K, μ_K, μ_s
θ, F_{max}

Find: Speed v at which bump force reaches F_{max}

<u>FBD</u>



<u>MAD</u>



$\underset{\sim}{N} = N\,\hat{e}_2$

$\underset{\sim}{W} = mg\sin(\theta)\,\hat{e}_1 - mg\cos(\theta)\,\hat{e}_2$

$\underset{\sim}{F_s} = -K\Delta\,\hat{e}_1$ ← where Δ is the compression of the spring

$\underline{F_f} = -\mu_K N\,\hat{e}_1$

Note: When spring is at its max allowable compression, Δ_{max}, the spring force is at its maximum, F_{max}

$$\Rightarrow F_{max} = K\Delta_{max} \Rightarrow \Delta_{max} = \frac{F_{max}}{K}$$

Newton

$$\hat{e}_2: \; N - mg\cos(\theta) = 0 \;\Rightarrow\; N = mg\cos(\theta)$$

$$\Rightarrow \; \underline{F}_f = -\mu_K mg\cos(\theta)\,\hat{e}_1$$

Work-Energy

$$U_{A\to B} = \cancel{T_B}^{\,0} - T_A$$

- Let state A be when spring is a distance d from the wall
- Let state B be when spring is at maximum compression Δ_{max}

Work by gravity

$$U_{A\to B}^{(grav)} = mg\,(d + \Delta_{max})\sin(\theta)$$

Work by spring

$$U_{A\to B}^{(spring)} = -\frac{1}{2}K\,\Delta_{max}^2$$

Work by friction

$$U_{A\to B}^{(frict)} = -\mu_K mg\cos(\theta)\left[d + \Delta_{max}\right]$$

Put it all together

$$mg(d+\Delta_{max})\sin(\theta) - \frac{1}{2}K\Delta_{max}^2 - \mu_k mg\cos(\theta)\left[d+\Delta_{max}\right] = -\frac{1}{2}mv^2$$

$$\Rightarrow \boxed{v = \left[-2g(d+\Delta_{max})\sin(\theta) + \frac{K}{m}\Delta_{max}^2 + \mu_k g\cos(\theta)\left[d+\Delta_{max}\right]\right]^{1/2}}$$

units $$\sqrt{\frac{L}{T^2}\cdot L + \frac{\cancel{M}}{T^2}\frac{1}{\cancel{M}}L^2 + \frac{L}{T^2}\cdot L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T}$$

✓