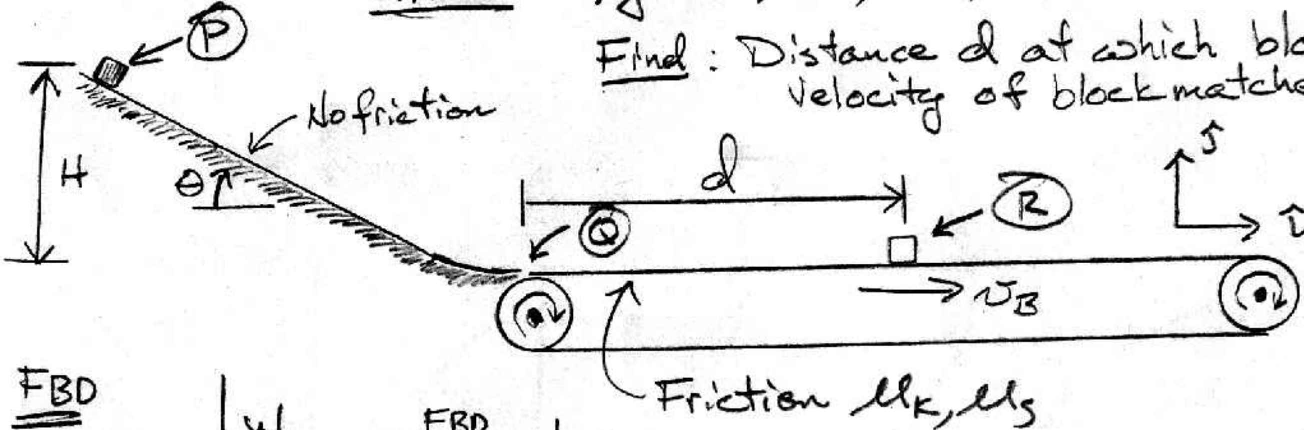
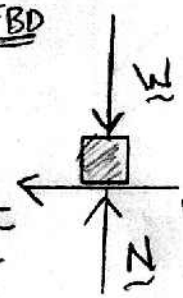
Given: $m, g, H, \theta, \mu_K, \mu_S, \quad v(0)=0$

Find: Distance d at which block velocity of block matches belt



FBD (on ramp)

FBD (on belt)



Might have friction in wrong direction

<u>Work-Energy</u> $U_{P\to Q} = T_Q - \cancel{T_P}$ → 0, since starts from rest

$$mgH = \tfrac{1}{2} m v_Q^2 \quad (1)$$

can solve for v_Q, $\Rightarrow v_Q = \sqrt{2gH}$

Note that $v_Q < v_B$ when $mgH < \frac{1}{2} m v_B^2$
& $v_Q > v_B$ when $mgH > \frac{1}{2} m v_B^2$ $\Big\}$ (*)

– Now go to conveyor belt

<u>Work-Energy</u> : $U_{Q\to R} = T_R - T_Q$

only friction performs work here

<u>Newton</u> $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$$\hat{j}: N - mg = 0 \Rightarrow N = mg$$

Friction: $\|\underset{\sim}{F}\| = \mu_K N = \mu_K mg$ ← Constant Force

Getting back to Work-Energy

$$\mho_{Q\to R} = T_R - T_Q$$

$$\mp \mu_K m g d = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_Q^2$$

$$= \frac{1}{2} m v_B^2 - m g H \quad \leftarrow \text{uses (1)}$$

So

$$\mp \mu_K m g d = \frac{1}{2} m v_B^2 - m g H \qquad (2)$$

Note that work done by friction may be positive or negative depending on which way the friction acts

→ Note that if $mgH < \frac{1}{2} m v_B^2$, then both sides of (2) are positive.

$$\Rightarrow \; + \mu_K \cancel{m} g d = \frac{1}{2} \cancel{m} v_B^2 - \cancel{m} g H \qquad (2a)$$

Work is positive. In this case $v_Q < v_B$ (see (*)) so friction speeds the block up.

$$\Rightarrow \boxed{d = \frac{\frac{1}{2} v_B^2 - g H}{\mu_K g}}$$

units $\quad \dfrac{L^2/T^2 - L/T^2 \cdot L}{L/T^2} = \dfrac{L^2/T^2}{L/T^2} = L \checkmark$

Now if $mgH > \frac{1}{2}mv_B^2$, then both sides of (2) are negative

$$\Rightarrow -\mu_k mgd = \frac{1}{2}mv_B^2 - mgH$$

Work is negative. In this case $v_Q > v_B$ (see (*)) so friction slows the block down

$$\Rightarrow \boxed{d = \frac{gH - \frac{1}{2}v_B^2}{\mu_k g}}$$

This answer has same units as previous answer

units ✓