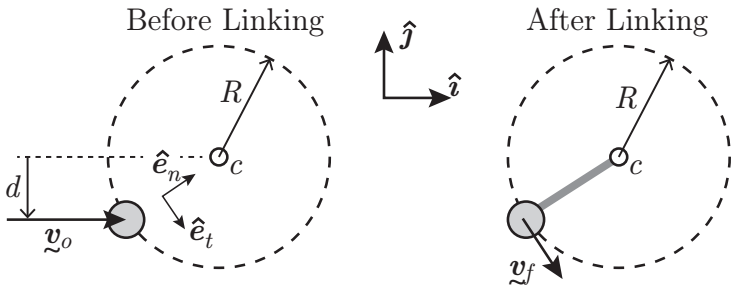
Link Analysis

Below, we show diagrams of Pokey just before and just after linking. Let R denote the radius of the link ring, and let m denote the mass of Pokey. Suppose that Pokey's velocity just before linking is $\underset{\sim}{\boldsymbol{v}}_o = v_o\,\hat{\boldsymbol{\imath}}$. As indicated in the figure, suppose that Pokey's trajectory intersects the link ring a distance d below the center, c.



1. Decompose the initial velocity vector $\underset{\sim}{\boldsymbol{v}}_0$ into components tangent and perpendicular to the link ring:

   $$\underset{\sim}{\boldsymbol{v}}_o = v_{ot}\,\hat{\boldsymbol{e}}_t + v_{on}\,\hat{\boldsymbol{e}}_n.$$

   Write the components v_{ot} and v_{on} in terms of quantities given: v_o, R, d, g, m.

2. Write a mathematical expression for $\underset{\sim}{\boldsymbol{H}}_c(t_o)$, the angular momentum vector of Pokey about center point c, just before linking. The vector should be in terms of quantities given.

3. Let $\underset{\sim}{\boldsymbol{v}}_f$ denote the velocity of Pokey just after linking. Because the link rod is rigid (does not stretch or compress), the velocity of Pokey is tangent to the circle: $\underset{\sim}{\boldsymbol{v}}_f = v_f\,\hat{\boldsymbol{e}}_t$.

   Write an expression for $\underset{\sim}{\boldsymbol{H}}_c(t_f)$, the angular momentum about point c just after linking, in terms of v_f, R, d, g, and m.

4. Draw a free body diagram of Pokey during the linking process. Don't forget about gravity.

5. Write the angular impulse - angular momentum principle about point c between time t_0 and t_f. Which of the forces participate in the angular impulse?

6. For the moment, assume that we can neglect the angular impulse due to gravity. Under this assumption, use the angular impulse - angular momentum principle to derive a relationship between v_f and v_o, along with other quantities given.

7. Use your result above to derive expressions for T_o and T_f, the kinetic energies of Pokey just before and just after linking. Both expressions should be written in terms of v_o, R, d, g, and m.

8. For what values of d is the most energy lost during linking? (Consider both positive and negative values of d in this and the next two questions.)

9. For what values of d is the least energy lost during linking?

10. Are there any values of d for which energy is gained during the linking process? If so, which?

Check assumption in Question 6. For this, use the following numbers for the parameters in the system: $v_o = 15\text{m/s}$, $R = 4\text{m}$, $d = 2.5\,\text{m}$, $g = 4\,\text{m/s}^2$, and $m = 2\text{kg}$.

11. In the Espoo world, linking happens over a time interval of about $\Delta t = 0.002$ seconds. Using the linear impulse - momentum principle, estimate the average force from the link rod during linking. (To do this, you may want to consider the a similar problem presented here [URL to come]).

12. Compare the rod force during linking to the force of gravity. Do you think the assumption in Question 6 is reasonable?

Generalizing Result. So far, we have studied the case in which the velocity of Pokey is perfectly horizontal. This is not true, in general.

13. Suppose that Pokey intersects the link ring at an angle other than horizontal as considered in Questions 1 through 10 as shown in the figure below. How can you generalize the results from Question 9. (You should not need to do any extra math here. Just think about what the previous results mean, qualitatively.)