A set V is a vector space if the following conditions are satisfied

1) There is an operation called "addition" such that

$$\vec{u} + \vec{v} \text{ is in } V$$

1a) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (commutative)

2) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (associative)

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2) V must contain a zero vector

$$\vec{v} + \vec{0} = \vec{v}, \text{ for all } \vec{v} \text{ in } V$$

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3) For $\vec{v}$ in V there has to be a vector we call $-\vec{v}$ such that

$$\vec{v} + (-\vec{v}) = \vec{0}$$

4) There is an operation called scalar multiplication such that if $\vec{v}$ is in V & there is a scalar s, then

$$s\vec{v} \text{ is in } V$$

Stuff not discussed in lecture

Furthermore, we require that for two scalars r & s

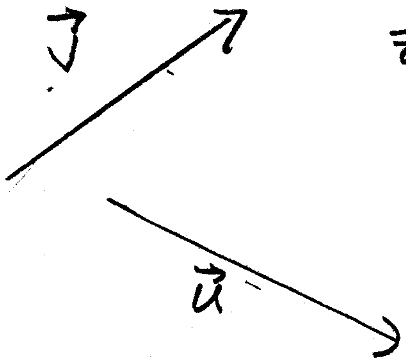
4a) $r(s\vec{v}) = (rs)\vec{v}$

4b) $\underset{\substack{\uparrow \\ \text{normal addition}}}{(r+s)}\vec{v} = r\vec{v} \underset{\substack{\uparrow \\ \text{vector addition}}}{+} s\vec{v}$

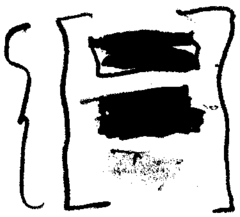
4c) $s(\vec{u}+\vec{v}) = s\vec{u}+s\vec{v}$

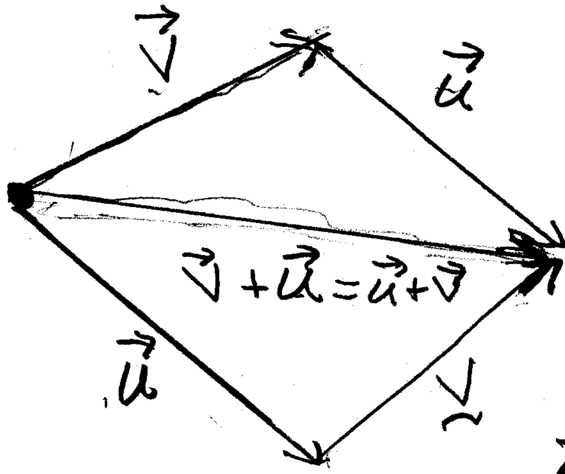
4d) $1\vec{v} = \vec{v}$ & $0\vec{v} = \vec{0}$

$$\begin{array}{ccccc} \vec{u} & & \vec{v} & & \vec{u}+\vec{v} \\ \begin{bmatrix} 6.28 \\ 1.28 \\ -4.0 \end{bmatrix} & + & \begin{bmatrix} 2.1 \\ 3.2 \\ 6.0 \end{bmatrix} & = & \begin{bmatrix} 8.38 \\ 4.43 \\ 2.0 \end{bmatrix} \end{array}$$



zero?

Colors  $0 \to 255 \quad \times 3$



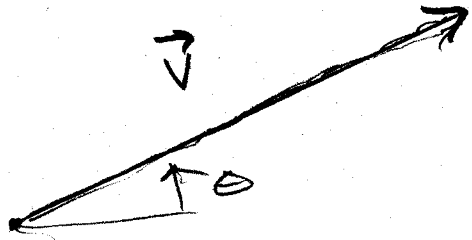
$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\boxed{\vec{u}+\vec{v} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}}$$

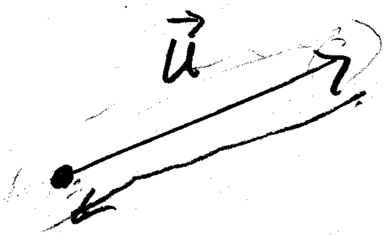
$$\begin{array}{ccccc} \vec{u} & & \vec{v} & & \vec{u}+\vec{v} \\ \begin{bmatrix} 2 \\ 1 \end{bmatrix} & + & \begin{bmatrix} 4 \\ 7 \end{bmatrix} & = & \begin{bmatrix} 9 \\ 5 \end{bmatrix} \end{array}$$

$$\cancel{\vec{u}+\vec{v} = \begin{bmatrix} u_1+v_2 \\ u_2+v_1 \end{bmatrix}}$$

$$\begin{bmatrix} 4 \\ 7 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 5 \\ 9 \end{bmatrix}}_{\vec{v}+\vec{u}} \qquad \vec{u}+\vec{v} \neq \vec{v}+\vec{u}$$

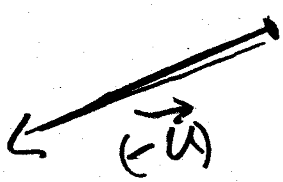


Zero? something w/ zero length & undefined direction



What is $(-\vec{u})$?

$\vec{u} + (-\vec{u})$



Take $s=2$

$2\vec{u}$