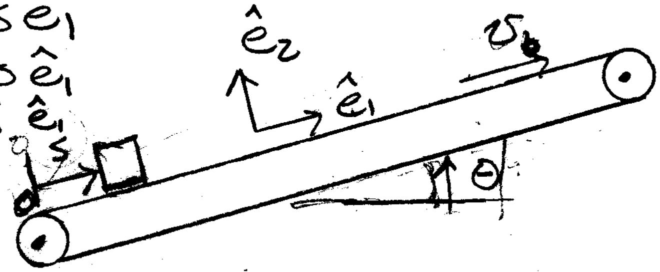
$\vec{P} = s\hat{e}_1$
$\vec{V} = \dot{s}\hat{e}_1$
$\vec{a} = \ddot{s}\hat{e}_1$



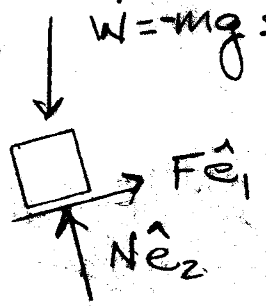
<u>Given</u>: $m, \theta, v_b, \mu_k, \mu_s, g$

<u>Find</u>: 1) t_{match} Time at which speed of block matches belt

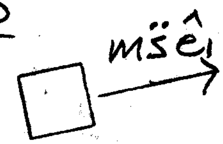
2) d_{match} Distance traveled @ $t = t_{match}$

3) Magnitude of friction force after t_{match}

<u>FBD</u> $\vec{W} = -mg\sin(\theta)\hat{e}_1 - mg\cos(\theta)\hat{e}_2$



<u>MAD</u>



<u>Newton's 2nd law</u> $\sum \vec{F} = m\vec{a}$

$\hat{e}_1$: $F - mg\sin(\theta) = m\ddot{s} \rightarrow \ddot{s} = \frac{F}{m} - g\sin(\theta)$ (*)

$\hat{e}_2$: $N - mg\cos(\theta) = 0 \rightarrow N = mg\cos(\theta)$

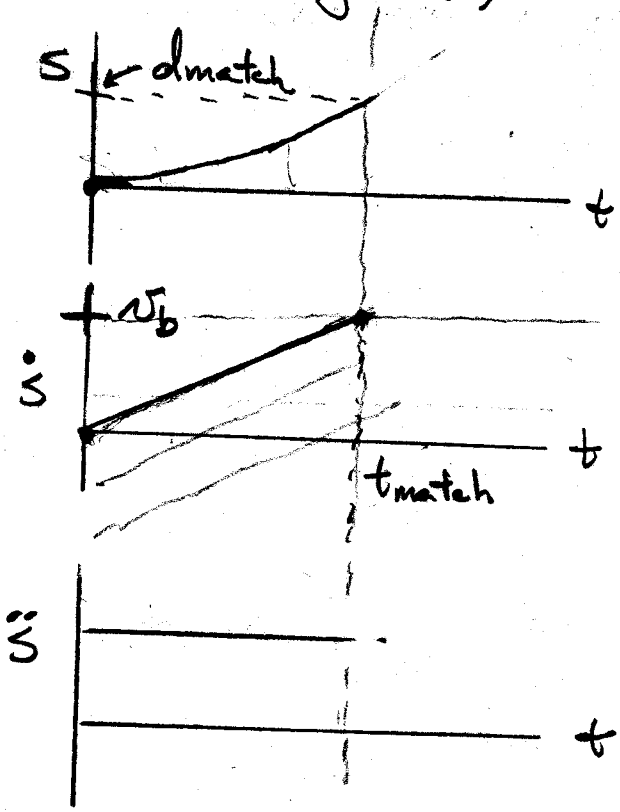
$\Rightarrow F = \mu_k N = \mu_k mg\cos(\theta)$

subst this into (*)

$\ddot{s} = \mu_k g\cos(\theta) - g\sin(\theta)$

$\ddot{s} = g(\mu_k\cos(\theta) - \sin(\theta))$

↑ positive constant since $F > mg\sin(\theta)$ because problem states that block travels uphill



$$\ddot{s} = g(\mu_k \cos(\theta) - \sin(\theta))$$

Integrate to find $\dot{s}$:

$$\dot{s} = \int g(\mu_k \cos(\theta) - \sin(\theta))\,dt = g(\mu_k \cos(\theta) - \sin(\theta))\,t + C_1$$

I.C.

$$\Rightarrow \dot{s}(0) = 0 = g(\mu_k \cos(\theta) - \sin(\theta))\,0 + C_1 \Rightarrow C_1 = 0$$

$$\Rightarrow \dot{s} = g(\mu_k \cos(\theta) - \sin(\theta))\,t \qquad (\#)$$

Speed of ice block matches speed of belt when...

$$\dot{s}(t_{match}) = g(\mu_k \cos(\theta) - \sin(\theta))\,t_{match} = v_b$$

$$\boxed{t_{match} = \frac{v_b}{(\mu_k \cos(\theta) - \sin(\theta))g}} > 0$$

check dims: $\dfrac{L/T}{(1_{\mu} + 1)\,L/T^2} = T$ ✓

Integrate (#) to find $s(t)$.

$$s(t) = \int g(\mu_k \cos(\theta) - \sin(\theta))\,t\,dt$$

$$= \frac{1}{2} g(\mu_k \cos(\theta) - \sin(\theta))\,t^2 + C_2$$

IC. $s(t{=}0) = \frac{1}{2} g(\mu_k \cos(\theta) - \sin(\theta))\,0^2 + C_2 = 0$

$$\Rightarrow C_2 = 0$$

$$s(t) = \frac{1}{2} g(\mu_k \cos(\theta) - \sin(\theta))\,t^2$$

Subst t_{match} into expression above

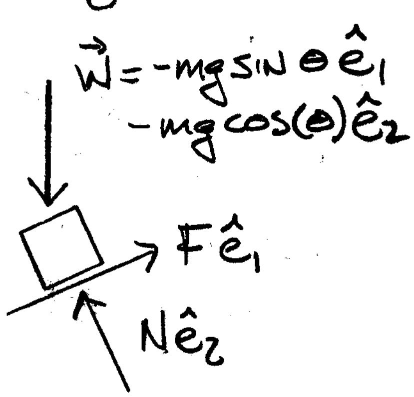
$$d_{match} = S(t_{match}) = \frac{g(\mu_k\cos(\theta)-\sin(\theta))}{2}\,\frac{v_b^2}{[(\mu_k\cos(\theta)-\sin(\theta))g]^2}$$

$$\boxed{d_{match} = \frac{v_b^2}{2g(\mu_k\cos(\theta)-\sin(\theta))}}$$

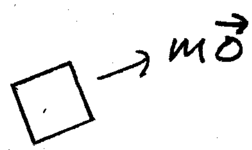
Check: $\dfrac{L^2/T^2}{L/T^2\,(1)} = L$

At time t_{match}, the two surfaces (block & belt) are no longer sliding. There is a possibility that they remain stuck together.

FBD



MAD



Static friction Force after t_{match}

Newton

$\hat{e}_1$: $F - mg\sin(\theta) = 0$ $\quad \boxed{F = mg\sin(\theta)}$

$F_{Break} > \mu_k N > mg\sin(\theta)$ $\quad \frac{ML}{T^2} = \text{Force}$ ✓